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Essays on local public goods and private schools

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ESSAYS ON LOCAL PUBLIC GOODS AND PRIVATE SCHOOLS

by

Muharrem Yesilirmak

An Abstract

Of a thesis submitted in partial fulfillment of the
requirements for the Doctor of Philosophy
degree in Economics
in the Graduate College of
The University of Iowa

May 2013

Thesis Supervisor: Professor B. Ravikumar

ABSTRACT

World is becoming more and more fiscally decentralized over time. Share of central government spending in total government spending declined from 75% to 65% between 1975 to 1995 in the world. Motivated by this, this thesis is concerned about two problems related to our current understanding of fiscally decentralized economies. In the first chapter, an explanation is given for the observed household income sorting pattern across municipalities where each municipality provides its own local public good. In the second chapter, an equilibrium existence result is provided for an economy where both local public schools and private schools coexist.

In the first chapter, I quantitatively explain the empirical household income distribution across municipalities. In the data, poor and rich households live together with varying fractions in all municipalities although there are large public expenditure differentials. To explain data, I construct a multi-community general equilibrium model at which heterogeneous income households probabilistically choose among communities where municipalities are comprised of several communities. The indivisibility in the choice set of households gives them the incentive to assign non-degenerate probabilities to each community which in turn gives rise to an income distribution resembling to that in data. The calibrated model is then used to analyze two public policies, uniform property tax rate and uniform housing supply across municipalities, with respect to their effects on income sorting.

The second chapter provides a median voter theorem for an economy where public and private schools coexist. Since households can opt out of public education, preferences over income tax rates are not single peaked leading possibly to nonexistence of majority voting equilibrium and decisive voter. Because of this, policy analysis of such economies proved difficult. To solve this nonexistence problem, I assume, consistently with empirical evidence, that private schools behave as monopolistically competitive firms with decreasing average costs over enrollment. In my model, there are a finite number of different quality private schools each having a different tuition. Public school spending is financed by income tax revenue collected from all households. The tax rate is determined by majority voting. I argue that preferences over tax rates are single peaked and therefore a majority voting equilibrium exists. Moreover median income household is the decisive voter. These results hold for any income distribution function and any finite number of private schools.

Abstract Approved: ______________________________

Thesis Supervisor

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Title and Department

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Graduate College
The University of Iowa
Iowa City, Iowa

CERTIFICATE OF APPROVAL

PH.D. THESIS

This is to certify that the Ph.D. thesis of

Muharrem Yesilirmak

has been approved by the Examining Committee for the thesis requirement for the Doctor of Philosophy degree in Economics at the May 2013 graduation.

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To my family

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CHAPTER 1
IMPERFECT INCOME SORTING IN AN ECONOMY WITH LOCAL PUBLIC GOODS

1.1 Introduction

Provision of public goods is becoming increasingly less centralized in the world over time. Using country-level data, Arzaghi and Henderson (2005) show that the share of central government expenditures in total expenditures decreased from 75% to 65% between 1975 and 1995 worldwide. For developed countries the decrease is from 57% to 46%. Standard theories of location choice (e.g., Tiebout (1956), Ellickson (1971) and Westhoff (1977)) in a fiscally decentralized economy imply households are perfectly sorted in income across municipalities based on their demand for the local public good. In U.S. data, although there are more than twofold differences in per pupil public spending levels across municipalities (Figure 1.1), households are not perfectly sorted in income. As illustrated in Figure 1.2, in the average state in 2000, 85% of the households in the richest municipality[1] and 20% of the households in the poorest municipality had incomes above the statewide median income. Moreover, a similar income sorting pattern also holds across census tracts (Figure 1.3) which are geographic units smaller than municipalities and differentiated mainly by median housing values (Figure 1.4). Ignoring poor households[2]

[1]The richest municipality has the highest median income and vice versa.

[2]The poor households are those with income below the statewide median income and vice versa.

in the rich municipality and rich households in the poor municipality, as previous theories do, biases the predictions of these models. For instance, per household public spending levels and housing prices would be underestimated in the poor municipality and overestimated in the rich municipality. Predictions regarding census tracts would also be biased. Motivated by these, this paper answers quantitatively the following questions in a general equilibrium model that is a simple generalization of Tiebout (1956): (1) Given large differentials in public spending levels, how can we explain the empirical household income sorting pattern across municipalities? (2) Given the large differentials in housing values, how can we explain the empirical household income sorting pattern across census tracts?

The income sorting pattern observed in data, which is called *imperfect income sorting*, has two components. The first component is household income mixing, which means that poor and rich households live in the same geographic areas. The second component is the disproportionate distribution of households; that is, poor households are disproportionately located in poorer municipalities and rich households are disproportionately located in richer municipalities. In order to explain imperfect income sorting, this paper takes advantage of the indivisibility nature of housing. In their choice set, heterogeneous income households are faced with a discrete number of house types in a discrete number of municipalities. This implies household preferences over house types are non-convex. This non-convexity induces households to randomize over house types. Households have access to a randomization device, called a lottery. Poorer households must win a higher prize in the lottery to buy expensive housing in a rich municipality, which happens with a lower probability. And to buy the same house, richer households must win a smaller prize, which happens with a higher probability. Relying on the law of large numbers thus implies the fraction of poor households is lower compared with richer households in a richer municipality, and the reverse happens in a poorer municipality. This way poor and rich households not only live together but the fractions also look similar to the data.

Imperfect income sorting is introduced for two income groups in Figure 1.2 whereas in the data there are sixteen different groups. Figures 1.5 through 1.8 plot the income distribution for the poorest and richest municipalities in all states. As seen in these figures, all income groups live in both types of municipalities. Moreover, income distributions are far from identical. In other words, the fraction of each income group differs across municipalities. Therefore, imperfect income sorting also holds after dividing income into more than two groups. One natural question is what percent of the statewide variance of income is due to the within-municipality variance of income? The values for this statistic, S, are provided in Table 1.1.[3] As seen from the table, on average the within-municipality variance of income accounts for 88% of the statewide variance of income. Table 1.2 provides the values of S for the census tracts. As seen, on average the within-census tract

[3]Detailed formula for S is provided in Appendix A.

variance of income accounts for 80% of the statewide variance of income.

Table 1.1: Income heterogeneity within municipalities

State	S	State	S
Arizona	0.91	Missouri	0.84
California	0.88	New York	0.88
Florida	0.89	North Carolina	0.92
Georgia	0.88	Pennsylvania	0.88
Illinois	0.85	Rhode Island	0.94
Massachusetts	0.88	Texas	0.88
Michigan	0.85	Virginia	0.81
Minnesota	0.86	Wisconsin	0.90

Table 1.2: Income heterogeneity within census tracts

State	S	State	S
Arizona	0.78	Missouri	0.81
California	0.77	New York	0.78
Florida	0.82	North Carolina	0.84
Georgia	0.79	Pennsylvania	0.81
Illinois	0.79	Rhode Island	0.85
Massachusetts	0.82	Texas	0.77
Michigan	0.81	Virginia	0.74
Minnesota	0.81	Wisconsin	0.86

To quantitatively account for these empirical observations, I build a general equilibrium model in which heterogenous households stochastically choose among heterogeneous census tracts. Each municipality is divided into several census tracts as in Ellickson (1979), Dunz (1985), and Nechyba (2003). Each census tract is associated with a particular house type.[4] Municipalities are heterogeneous with respect to property tax rates, local public spending per household, value, and the fixed stock of several house types. There is a continuum of households that are heterogeneous with respect to income and derive utility from consumption, housing, and public spending per household. They choose a lottery, along the lines of Prescott and Townsend (1984), that is a probability distribution over prizes for each house type in each municipality. Fair odds gambling applies so that each lottery has zero expected gain or loss. Taking as given exogenously economy-wide income distribution, municipal property tax rate, and the stock of each house type, the model endogenously determines income distribution, the value of each house type, and local public spending per household in each municipality.

The mortgage market provides a real-life example for lotteries. According to the Census Bureau's Residential Finance Survey, roughly 97% of all housing units were purchased through mortgage loans in 2001. Households pay an application fee for each mortgage credit application, and there is uncertainty regarding the approval of the application. According to Mortgage Bankers Association, between 30% and 40% of mortgage applications are denied. Households that are rejected lose

[4]Throughout the paper "house type" and "census tract" are used interchangeably.

some part of their income whereas households that are approved receive positive credit on top of their income. Rejected households buy cheap houses in the poor municipality and approved households buy expensive houses in the rich municipality. Poor households apply for much higher credit than rich households which decreases the probability of being approved. Therefore, the probability that a poor household will be approved is lower than a rich household's probability of being approved. This explains the fraction of different income groups in different municipalities.

With the calibrated model at hand which is consistent with imperfect income sorting, two policy questions are posed: (1) How much does property tax competition affect the sorting of households across municipalities? (2) How much would income sorting change if the supply of each house type were equalized across municipalities? For the first question, I conduct a counterfactual experiment that exogenously sets the mean benchmark residential property tax rate as the new tax rate in each municipality. Eliminating tax differentials causes rich households living in poor municipalities to migrate to rich municipalities. This increases housing prices in rich municipalities, which in turn causes poor households to relocate to poor municipalities. Therefore, income sorting increases by 17% under the first policy experiment. To answer the second question, I exogenously set the mean supply of a particular house type in the whole economy as the new supply for that type in each municipality. As a result, the supply of housing becomes identical across municipalities under the experiment. Compared with the benchmark, the supply of low-quality housing is increased in the rich municipality and the supply of high-quality housing is increased in the poor municipality. This policy creates incentives through the housing market for poor and rich households to live in the same municipality. As a fulfillment of these intuitive expectations, under this policy income sorting in the society decreases by 56%.

The paper is organized as follows: Section 2 reviews the previous literature. Section 3 outlines the model. Section 4 calibrates the model. Section 5 compares the model's predictions with respect to empirical facts. Section 6 reports results of the computational experiments. Section 7 concludes.

1.2 Previous literature

In this section, I review the papers most closely related to my paper which are those by Epple and Platt (1998), Dunz (1985), Nechyba (1999), and McFadden (1978).

In Epple and Platt (1998), each household is heterogeneous with respect to both income and preference for housing, where housing is a perfectly divisible commodity. Income and preference for housing are positively correlated. Among rich households, the fraction of high-preference types is higher than low-preference types. Similarly, among poor households, the fraction of low-preference types is higher than high-preference types. Households self-select themselves across municipalities. Poor municipalities have lower lump-sum public transfers, lower housing tax rates, and lower housing prices, whereas the reverse is true in rich municipalities. In equilibrium, poor and rich households with similar preferences for housing live in the same municipality. This, combined with assumptions on the joint distribution of income and preference types, yields the correct fractions.[5] This may not be desirable in policy analysis, where the fraction of households living in a particular municipality is expected to adjust endogenously with policy changes, since households may be heterogeneous in several other characteristics. Moreover, in equilibrium there is perfect sorting with respect to income after conditioning on the preference parameter for housing. Under Cobb-Douglas preferences, this parameter is equivalent to the rent share in income. Therefore, households are perfectly sorted in income across municipalities after controlling for the rent share in income. This implication is at odds with the data. Figure 1.9 plots the percentage of households whose income is above the state median income for the poorest and richest municipalities in each state. The sample of households is restricted to those whose rent share in income less than 14% which is roughly 50% of all households. As seen in the figure, conditional on rent share in income, households are far from perfectly sorted by income across municipalities.

In Dunz (1985) and Nechyba (1999),[6] each municipality is divided into several census tracts with each census tract corresponding to a different house type. Each municipality determines its own public spending level. In the Dunz-Nechyba model,

[5]Epple and Sieg (1999), Schmidheiny (2006a), and Schmidheiny (2006b) also rely on a similar mechanism.

[6]A similar model is used in Nechyba (2000) and Nechyba (2003)

households are heterogeneous with respect to both income and the initial endowment of house type. Therefore, same-income households start with different wealth levels, where the wealth of a household is defined as the sum of income and the value of the house endowment. Households then trade their houses to maximize utility. Household utility depends on consumption, housing quality, and public spending per household. In this model, same-income households buy different quality houses in different municipalities. This mechanism creates imperfect income sorting across census tracts and municipalities. However, this model also implies perfect sorting with respect to wealth across both municipalities and census tracts. In other words, support of the distribution of wealth does not overlap across municipalities or census tracts. This is illustrated in Figure 1.26 for the case of two municipalities and two different census tracts in each municipality, creating a total of four alternatives. In the figure, lines 1 and 3 denote census tracts in municipality one, and lines 2 and 4 denote census tracts in municipality two as before. As seen, households whose wealth level is between $[0, w_1]$ choose census tract 1, between $[w_1, w_2]$ choose census tract 2, between $[w_2, w_3]$ choose census tract 3, and those with wealth between $[w_3, \infty)$ choose census tract 4. This implies $[0, w_1] \bigcup [w_2, w_3]$ live in municipality one and $[w_1, w_2] \bigcup [w_3, \infty)$ live in municipality two. One natural question to ask is whether there is perfect sorting with respect to wealth in the data when wealth is as defined above. Figure 1.27 plots the wealth of households against the percentage of municipalities in which households of a particular wealth level are living.[7] If a particular wealth level households are observed in all municipalities then the corresponding value on the y-axis would be 100%. Figure 1.28 plots the same situation for census tracts. These figures suggest a considerable amount of imperfect sorting in wealth across both municipalities and census tracts.

In McFadden (1978), same-income households receive different preference shocks to their utility.[8] Given that these shocks are distributed with extreme value distribution, it can be shown that for a particular income household, the probability of choosing a particular location is equal to the ratio of indirect utility received from that location to the sum of indirect utilities across all locations. This ratio is called the logit function and follows from Luce (1959)'s axiom. The probability of assigning a particular income household to a particular alternative is positive unless the utility received from that alternative is zero. Because of this, logit framework predicts that middle and high-income (or wealth) households live in all municipalities or census tracts, which is at odds with the data presented in Figures 1.13, 1.27, and 1.28. Moreover, Debreu (1960) and McFadden (1973) argue that logit framework is subject to the so-called duplicates effect, which may bias the results of policy experiments.

[7]Please see Appendix A for more information on data used in Figure 1.27.

[8]Ellickson (1977), Anas (1980), Bayer, McMillan, and Rueben (2005), Ferreyra (2007), and Luk (1993) build on the same idea.

1.3 Model

Imagine a static environment with $M \geq 2$ municipalities, $H \geq 1$ different house types in each municipality, and a continuum of households over $[0, 1]$. Therefore, in total there are $M \times H$ different house types denoted with mh. Each municipality is heterogeneous with respect to housing property tax rates, local public spending per household, net state aid, house value, and fixed stock of several house types. The value of each house type and local public spending per household are determined endogenously in equilibrium, whereas the property tax rate, net state aid, and stock of each house type are exogenously given. Households are heterogeneous with respect to income and derive utility from consumption, housing quality, and public spending per household. Households rent their house and they are perfectly mobile with zero mobility cost. Households have access to lotteries supplied by perfectly competitive, risk-neutral firms. Each lottery is a vector of probabilities and payoffs for each house type in each municipality. There is a local government in each municipality that collects property taxes and spends the whole amount on locally provided public good.

1.3.1 Preferences

Households have identical preferences defined over the commodity space,

$$X = \{((c_{mh}, q_{mh}, E_{mh}, \pi_{mh})_{h=1}^{H})_{m=1}^{M} \in \Re_{+}^{4\times M\times H} : \sum_{m=1}^{M}\sum_{h=1}^{H}\pi_{mh} = 1\}$$

where c_{mh}, q_{mh}, and E_{mh} represent, respectively, consumption of the numeraire good, quality of the house, and per household public spending in alternative mh, which is realized with probability π_{mh}.

Having defined commodity space, preferences are represented by an expected utility form as follows:

$$\sum_{m=1}^{M}\sum_{h=1}^{H} U(c_{mh}, q_{mh}, E_{mh})\pi_{mh},$$

where the Bernoulli utility function $U(\cdot)$ satisfies the following assumptions:

Assumption 1. *$U(c_{mh}, q_{mh}, E_{mh})$ is twice continuously differentiable in c_{mh} with $U_{11}(c_{mh}, q_{mh}, E_{mh}) < 0$ for any mh and $c_{mh} > 0$.*

Assumption 2. *$U_1(c_{mh}, q_{mh}, E_{mh}) > 0$ for any mh and $c_{mh} > 0$.*

Assumption 3. *$\lim_{c_{mh} \to 0} U_1(c_{mh}, q_{mh}, E_{mh}) = -\infty$ for any mh.*

Assumption 4. *$\lim_{c_{mh} \to \infty} U_1(c_{mh}, q_{mh}, E_{mh}) = 0$ for any mh.*

Assumption 5. *For two alternatives $mh \neq m'h'$, if $U(c, q_{mh}, E_{mh})$ is strictly greater than $U(c, q_{m'h'}, E_{m'h'})$ at a specific consumption level $c > 0$, then $U(c, q_{mh}, E_{mh})$ is also strictly greater than $U(c, q_{m'h'}, E_{m'h'})$ for any $c > 0$.*

Assumption 6. *For two alternatives $mh \neq m'h'$, if $U(c, q_{mh}, E_{mh})$ is strictly greater than $U(c, q_{m'h'}, E_{m'h'})$ for any $c > 0$, then $U_1(c, q_{mh}, E_{mh})$ is also strictly greater than $U_1(c, q_{m'h'}, E_{m'h'})$ for any $c > 0$.*

Assumptions $1-4$ are standard but Assumptions 5 and 6 require more explanation. To graphically explain these assumptions, Figure 1.29 plots utility as a function of numeraire consumption for house types mh and $m'h'$. Utility is assumed to be higher under mh compared with $m'h'$ for any consumption level. As also seen from the slopes of the two curves, the marginal utility of consumption is higher under alternative mh for any $c > 0$. These assumptions are required to guarantee single crossing between two consecutive utility functions, as explained in more detail below.

1.3.2 Endowments

Households are heterogeneous with respect to exogenous receipts of income y measured in terms of numeraire consumption. Income is distributed according to a cumulative distribution function $F(\cdot)$ with support $\Re_+$.

1.3.3 Lotteries

The lottery is modeled as in Marshall (1984), Prescott and Townsend (1984), Bergstrom (1986), Garratt and Marshall (1994), and Cole and Prescott (1997). Each lottery is a vector of probabilities $((\pi_{mh})_{h=1}^{H})_{m=1}^{M}$ and prizes $((z_{mh})_{h=1}^{H})_{m=1}^{M}$ for each house type. Prizes are allowed to take both positive and negative values and are measured in terms of the consumption good. Moreover, each lottery is assumed to behave like an actuarially fair gamble, which means there is zero expected gain or loss. Therefore, for each lottery,

$$\sum_{m=1}^{M}\sum_{h=1}^{H} z_{mh}\pi_{mh} = 0. \tag{1.1}$$

This condition implies that aggregate receipts equal the aggregate value of prizes distributed. In other words, the lottery market clears in the aggregate.

Lotteries are supplied by perfectly competitive, risk-neutral firms. Supplier firms do not care about the probabilities and prizes involved in a lottery since there is zero expected gain or loss from each. Each lottery can be thought of as a financial contract between households and suppliers. Both parties commit ex ante on prizes for each state mh. Depending on the realization of the state, each household receives either a positive or a negative prize.

1.3.4 Housing market

There are $M \times H$ different house types in the model. Each house type has a different quality parameter, denoted by q_{mh}. Quality of a house q_{mh} captures both the housing services received from the house and neighborhood-specific amenities other than municipal public spending per household. The supply of house type mh is denoted by $\mu_{mh} > 0$, which is a fixed exogenous number. The value of house type mh denoted by p_{mh} is determined so as to equate the household demand to supply.

1.3.5 Household's decision problem

Given housing property tax rates $\{\tau_m\}_{m=1}^{M}$, per household public spending levels $\{E_m\}_{m=1}^{M}$, house values, and qualities $\{\{p_{mh}, q_{mh}\}_{h=1}^{H}\}_{m=1}^{M}$ for each house type, the household's problem with income y is,

$$\max_{\substack{\{\{c_{mh}\}_{h=1}^{H}\}_{m=1}^{M} \\ \{\{z_{mh}\}_{h=1}^{H}\}_{m=1}^{M} \\ \{\{\pi_{mh}\}_{h=1}^{H}\}_{m=1}^{M}}} \sum_{m=1}^{M}\sum_{h=1}^{H} U(c_{mh}, q_{mh}, E_m)\pi_{mh} \tag{1.2}$$

subject to

$$c_{mh} + r_{mh} + \tau_m p_{mh} = y + z_{mh} \qquad \forall(m,h)$$

$$\sum_{m=1}^{M}\sum_{h=1}^{H} z_{mh}\pi_{mh} = 0$$

$$\sum_{m=1}^{M}\sum_{h=1}^{H} \pi_{mh} = 1$$

$$\pi_{mh} \in [0,1] \;\; \forall(m,h)$$

$$c_{mh} \geq 0 \;\; \forall(m,h),$$

where r_{mh} denotes the annual rent for house type mh and it is determined by a no-arbitrage condition:

$$p_{mh} = \sum_{t=0}^{\infty} \frac{r_{mh}}{(1+\rho)^t}$$

or equivalently

$$r_{mh} = \frac{\rho}{1+\rho} p_{mh}, \tag{1.3}$$

where ρ is the real annual interest rate given exogenously.

A household's total income in state mh consists of annual income y and lottery prize z_{mh}. Total income is spent on numeraire consumption, house rent, and housing property tax. Each household is assumed to rent at most one unit of a house type mh. The price of the numeraire consumption good is normalized to 1.

In order to ease the understanding of the household's problem (1.2), I reformulate it as a two-step optimization problem as in Marshall (1984), Bergstrom (1986), and Garratt and Marshall (1994). In the first step, the household solves the following problem given $\{\{\pi_{mh}\}_{h=1}^{H}\}_{m=1}^{M}$ along with y, $\{\tau_m\}_{m=1}^{M}$, $\{E_m\}_{m=1}^{M}$, and $\{\{q_{mh}\}_{h=1}^{H}\}_{m=1}^{M}$:

$$\max_{\substack{\{\{c_{mh}\}_{h=1}^{H}\}_{m=1}^{M} \\ \{\{z_{mh}\}_{h=1}^{H}\}_{m=1}^{M}}} \sum_{m=1}^{M}\sum_{h=1}^{H} U(c_{mh}, q_{mh}, E_m)\pi_{mh} \tag{1.4}$$

subject to

$$c_{mh} + r_{mh} + \tau_m p_{mh} = y + z_{mh} \qquad \forall(m,h)$$
$$\sum_{m=1}^{M}\sum_{h=1}^{H} z_{mh}\pi_{mh} = 0$$
$$c_{mh} \geq 0 \;\; \forall(m,h).$$

The above problem (1.4) gives us the optimal consumption $\{\{c_{mh}^{y*}(\ell)\}_{h=1}^{H}\}_{m=1}^{M}$ and optimal lottery prizes $\{\{z_{mh}^{y*}(\ell)\}_{h=1}^{H}\}_{m=1}^{M}$ as a function of $\ell = \{\{\pi_{mh}\}_{h=1}^{H}\}_{m=1}^{M}$. In the second stage, the household chooses the probabilities that maximize expected utility:

$$\max_{\ell = \{\{\pi_{mh}\}_{h=1}^{H}\}_{m=1}^{M}} \sum_{m=1}^{M}\sum_{h=1}^{H} U(c_{mh}^{*}(\ell), q_{mh}, E_m)\pi_{mh} \tag{1.5}$$

subject to

$$\sum_{m=1}^{M}\sum_{h=1}^{H} \pi_{mh} = 1$$
$$\pi_{mh} \in [0,1] \;\; \forall(m,h).$$

Therefore, solving the household's problem (1.2) yields the probabilistic assignment of households to house types. Aggregating over house types in a municipality gives us the distribution of households in that municipality. The probability density function of income is denoted by $f_m(y) : \Re_+ \rightarrow [0,1]$, and the probability mass function of house values is denoted by $g_m(p) : \Re_+ \rightarrow [0,1]$ for municipality m. In words, $f_m(y)$ is the proportion of households with income y, and $g_m(p)$ shows the proportion of houses with value p in municipality m.

1.3.6 Local government and public spending

Given $g_m(p)$, the local government determines the public spending per household in municipality m as follows:

$$E_m = \tau_m \sum_{h=1}^{H} p_{mh} \cdot g_m(p_{mh}) + NSA_m. \tag{1.6}$$

The first term on the right-hand side of (1.6) is the per household residential property tax revenue in municipality m. And NSA_m denotes the per household net state aid to municipality m, which is exogenously given.

1.3.7 Remarks

Definition 1. *The user cost of a house type mh is defined as $s_{mh} = r_{mh} + \tau_m p_{mh}$.*

1. Households always randomize between at most two house types. In other words, a household with a particular income level will assign positive probability to at most two house types. To illustrate this point, assume there are two municipalities and two different house types in each municipality creating a total of four alternatives denoted 11, 21, 12, and 22. The indirect utility

as a function of income is plotted in Figure 1.22 for each house type. Figure 1.30 demonstrates which income interval randomizes between which two alternatives. As seen, households with income in $[s_{11}, y_1]$ will choose house type 11 for certain since this choice gives them the highest indirect utility. Similarly, households with income in $[y_4, \infty]$ will choose house type 22 for certain. On the other hand, households with income in $[y_1, y_2]$ will find it more optimal to randomize between alternatives 11 and 21. Similar arguments hold for households with income in $[y_2, y_3]$ and $[y_3, y_4]$. Randomizing between only two alternatives is also the case in Kalai and Megiddo (1980), Marshall (1984), Bergstrom (1986), and Garratt and Marshall (1994).

2. In this remark I want to explain why the model presented above produces imperfect income sorting across municipalities and census tracts. Indirect utility from each house type as a function of income is illustrated in Figure 1.22. User costs are denoted by s_{11}, s_{21}, s_{12}, and s_{22}. Therefore, user costs are lower in municipality 1. Since there are a discrete number of house types, the indirect utility function of the household (see Figure 1.23) is the upper envelope of the indirect utility functions in Figure 1.22.[9] Notice the kinks at the income levels y_1, y_2, and y_3. Non-convexity of the indirect utility function gives households the incentive to randomize. For example, consider the household with income

[9]This feature of indirect utility function under indivisibility is also argued in Friedman and Savage (1948) and Ng (1965).

level y_4 in Figure 1.24. This household receives utility V_1 under no randomization. When lotteries are available, this household randomizes between house types 12 and 22 by choosing prizes z_{12}, z_{22} and probabilities p_{22}, $p_{12} = 1 - p_{22}$. The resulting utility level is V_2, which is greater than V_1. It should be noted that p_{12} is greater than p_{22}. Now consider a richer household with income y_5 greater than y_4. From Figure 1.25, it is clear that the probability assigned by this household to alternative 12 is smaller than the probability assigned to alternative 22. In other words, $p^{'}_{12} < p^{'}_{22} = 1 - p^{'}_{12}$. Therefore, poorer (richer) households assign higher probability to poorer (richer) municipalities or census tracts.

3. Tiebout (1956) model is a special case of the model above when lottery probabilities, π_{mh}, are allowed to take values of only 0 or 1. This understanding of Tiebout's model is consistent with the assumptions listed in Tiebout (1956), which are as follows:

   (a) Households are fully mobile and choose the municipality that best satisfies their preferences.

   (b) Households have full information about tax and public expenditures in all municipalities.

   (c) Labor market differences across municipalities do not affect a household's decision.

   (d) Public good provision is completely local.

(e) There is an optimum municipality population defined in terms of a fixed factor, such as land area combined with a set of zoning laws.

(f) Optimum population is reached through an economic force.

The first four assumptions are already clear. The last two need more explanation. Assumptions e and f are captured by the housing market in my model. The fixed supply of house types, together with the inability to rent more than one house type, corresponds to assumption e above. Moreover, the value of a house type is determined by the housing market clearing condition for that type. This price mechanism corresponds to the economic force mentioned in assumption f.

1.3.8 Equilibrium[10]

An equilibrium is a collection of distribution functions $\{f_m(y), g_m(p^*)\}_{m=1}^M$, per household public spending levels $\{E_m^*\}_{m=1}^M$, housing values and rents $\{\{p_{mh}^*, r_{mh}^*\}_{h=1}^H\}_{m=1}^M$, housing property tax rates $\{\tau_m\}_{m=1}^M$, per household net state aid $\{NSA_m\}_{m=1}^M$, housing supplies $\{\{\mu_{mh}\}_{h=1}^H\}_{m=1}^M$, and optimal decisions $\{\{c_{mh}^*, \pi_{mh}^*, z_{mh}^*\}_{h=1}^H\}_{m=1}^M$ for each household such that:

i) $\{\{c_{mh}^*, \pi_{mh}^*, z_{mh}^*\}_{h=1}^H\}_{m=1}^M$ solves the decision problem of the household given income, $\{E_m^*\}_{m=1}^M)$, $\{\{p_{mh}^*\}_{h=1}^H\}_{m=1}^M$, and $\{\tau_m\}_{m=1}^M$.

ii) There is no arbitrage in the housing market, i.e. equation (1.3) holds.

[10]Equilibrium values are denoted with an asterisk (*) hereafter.

iii) The equilibrium distributions $\{f_m(y), g_m(p^*)\}_{m=1}^M$ and $\{E_m^*\}_{m=1}^M$ are consistent with the households' optimal decisions.

iv) The housing market clears for each alternative mh:

$$\mu_{mh} = \int \pi_{mh}^* dF(y)$$

v) The local government budget balances in each municipality m:

$$E_m = \tau_m \sum_{h=1}^{H} p_{mh} \cdot g_m(p_{mh}) + NSA_m.$$

1.3.9 Characterization of equilibrium

Lemma 1. *For any two alternatives mh and $m'h'$, if $U(c, q_{mh}, E_{mh}^*)$ is strictly greater than $U(c, q_{m'h'}, E_{m'h'}^*)$ for any $c > 0$, then in any equilibrium $s_{mh}^* > s_{m'h'}^*$.*

Proof. Assume to the contrary that $s_{mh}^* < s_{m'h'}^*$. Given that $U(c, q_{mh}, E_{mh}^*) > U(c, q_{m'h'}, E_{m'h'}^*)$ for any $c > 0$, then any household will prefer alternative mh over $m'h'$. This means demand for alternative house type $m'h'$ is zero, which contradicts the market-clearing condition for $m'h'$ since $\mu_{m'h'} > 0$ by assumption. □

Lemma 1 simply states that an alternative that gives higher utility compared with another alternative at all consumption levels should have higher user cost in equilibrium.

Definition 2. *Two house types mh and $m'h'$ are called "consecutive" if there does not exist a third alternative $m''h''$ such that either $s_{mh}^* < s_{m''h''}^* < s_{m'h'}^*$ or $s_{m'h'}^* < s_{m''h''}^* < s_{mh}^*$ holds in equilibrium.*

Proposition 1. *For any two consecutive house types mh and $m'h'$, there exists a set of households with positive measure who assign positive probability to each house type in equilibrium.*

Proof. Without loss of generality, let us assume that $U(c, q_{mh}, E^*_{mh})$ is greater than or equal to $U(c, q_{m'h'}, E^*_{m'h'})$ for any $c > 0$. Then Lemma 2 implies $s^*_{mh} > s^*_{m'h'}$. This, combined with assumptions 1-6 implies the existence of a unique income level $\widehat{y}$ at which:

$$V(\widehat{y}, s^*_{mh}, q_{mh}, E^*_{mh}) = V(\widehat{y}, s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) \tag{1.7}$$

The first claim is that this household with income $\widehat{y}$ assigns positive probability to both alternatives mh and $m'h'$. To prove this, assume to the contrary that this agent chooses either alternative with probability 1. Without loss of generality, let us assume alternative mh is chosen with probability 1. The indirect utility of this agent is equal to $V(\widehat{y}, s^*_{mh}, q_{mh}, E^*_{mh})$. Now let us compare this with the indirect utility at which $\pi_{mh} = 0.5$ and $z^*_{mh} = \varepsilon$. By the fair odds gambling condition (1.1), $z^*_{m'h'} = -\varepsilon$. The resulting indirect utility is:

$$0.5[V(\widehat{y} + \varepsilon, s^*_{mh}, q_{mh}, E^*_{mh}) + V(\widehat{y} - \varepsilon, s^*_{m'h'}, q_{m'h'}, E^*_{m'h'})].$$

Given the supposition:

$$0.5[V(\widehat{y} + \varepsilon, s^*_{mh}, q_{mh}, E^*_{mh}) + V(\widehat{y} - \varepsilon, s^*_{m'h'}, q_{m'h'}, E^*_{m'h'})] < V(\widehat{y}, s^*_{mh}, q_{mh}, E^*_{mh}),$$

or equivalently using (1.7),

$$
\begin{aligned}
&V(\widehat{y}+\varepsilon, s^*_{mh}, q_{mh}, E^*_{mh}) - V(\widehat{y}, s^*_{mh}, q_{mh}, E^*_{mh}) + \\
&V(\widehat{y}-\varepsilon, s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) - V(\widehat{y}, s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) < 0.
\end{aligned} \tag{1.8}
$$

Using $s^*_{mh} > s^*_{m'h'}$, Assumption 1 and Assumption 6, together with a low enough ε implies:

$$
V_1(\widehat{y}, s^*_{mh}, q_{mh}, E^*_{mh}) - V_1(\widehat{y}, s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) > 0,
$$

which contradicts (1.8). Therefore, the agent with income $\widehat{y}$ assigns positive probability to both alternatives mh and $m'h'$.

Given this finding, the strictly concave first-stage and second-stage optimization problems, (1.4) and (1.5), yield unique interior solutions for a household with income $\widehat{y}$. The first-order conditions of these problems are sufficient and given by:

$$
U_1(\widehat{y} + z^{\widehat{y}*}_{mh} - s^*_{mh}, q_{mh}, E^*_{mh}) = U_1(\widehat{y} + z^{\widehat{y}*}_{m'h'} - s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) \tag{1.9}
$$

$$
\begin{aligned}
U(c^{\widehat{y}*}_{mh}, q_{mh}, E^*_{mh}) + U_1(c^{\widehat{y}*}_{mh}, q_{mh}, E^*_{mh}) \left[\pi^{\widehat{y}*}_{mh} \frac{\partial c^{\widehat{y}*}_{mh}}{\partial \pi_{mh}} + \pi^{\widehat{y}*}_{m'h'} \frac{\partial c^{\widehat{y}*}_{m'h'}}{\partial \pi_{mh}} \right] &= \\
U(c^{\widehat{y}*}_{m'h'}, q_{m'h'}, E^*_{m'h'}) + U_1(c^{\widehat{y}*}_{m'h'}, q_{m'h'}, E^*_{m'h'}) \left[\pi^{\widehat{y}*}_{mh} \frac{\partial c^{\widehat{y}*}_{mh}}{\partial \pi_{m'h'}} + \pi^{\widehat{y}*}_{m'h'} \frac{\partial c^{\widehat{y}*}_{m'h'}}{\partial \pi_{m'h'}} \right]&.
\end{aligned} \tag{1.10}
$$

Budget constraint implies that for any y and $i, j \in \{mh, m'h'\}$,

$$
\frac{\partial c^{y*}_i}{\partial \pi_j} = \frac{\partial z^{y*}_i}{\partial \pi_j}. \tag{1.11}
$$

Also, differentiating the fair odds gambling condition with respect to probabilities implies for any y that:

$$z^{y*}_{mh} + \pi^{y*}_{mh}\frac{\partial z^{y*}_{mh}}{\partial \pi_{mh}} + \pi^{y*}_{m'h'}\frac{\partial z^{y*}_{m'h'}}{\partial \pi_{mh}} = 0 \tag{1.12}$$

$$z^{y*}_{m'h'} + \pi^{y*}_{mh}\frac{\partial z^{y*}_{mh}}{\partial \pi_{m'h'}} + \pi^{y*}_{m'h'}\frac{\partial z^{y*}_{m'h'}}{\partial \pi_{m'h'}} = 0. \tag{1.13}$$

Using equations (1.9), (1.11), (1.12) and (1.13), equation (1.10) reduces to

$$U(\widehat{y} + z^{\widehat{y}*}_{mh} - s^*_{mh}, q_{mh}, E^*_{mh}) = U(\widehat{y} + z^{\widehat{y}*}_{m'h'} - s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) + U_1(\widehat{y} + z^{\widehat{y}*}_{mh} - s^*_{mh}, q_{mh}, E^*_{mh})(z^{\widehat{y}*}_{mh} - z^{\widehat{y}*}_{m'h'}). \tag{1.14}$$

Now let us consider an arbitrary household with income $y \in [\widehat{y} + z^{\widehat{y}*}_{m'h'}, \widehat{y} + z^{\widehat{y}*}_{mh}]$. I guess that the optimal prizes chosen by this agent in states mh and $m'h'$ are $z^{y*}_{mh} = \widehat{y} + z^{\widehat{y}*}_{mh} - y$ and $z^{y*}_{m'h'} = \widehat{y} + z^{\widehat{y}*}_{m'h'} - y$, respectively. Since household problems (1.4) and (1.5) are strictly concave, this guess must satisfy the following first-order conditions:

$$U_1(y + z^{y*}_{mh} - s^*_{mh}, q_{mh}, E^*_{mh}) = U_1(y + z^{y*}_{m'h'} - s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) \tag{1.15}$$

$$U(y + z^{y*}_{mh} - s^*_{mh}, q_{mh}, E^*_{mh}) = U(y + z^{y*}_{m'h'} - s^*_{m'h'}, q_{m'h'}, E^*_{m'h'}) + U_1(y + z^{y*}_{mh} - s^*_{mh}, q_{mh}, E^*_{mh})(z^{y*}_{mh} - z^{y*}_{m'h'}). \tag{1.16}$$

These conditions are equal to (1.9) and (1.14), respectively at the guess. This verifies the guess. The associated probabilities chosen by this household with income y can be found from the fair odds gambling condition as follows:

$$\pi_{mh} = \frac{y - \widehat{y} - z^{\widehat{y}*}_{m'h'}}{z^{\widehat{y}*}_{mh} - z^{\widehat{y}*}_{m'h'}}$$

$$\pi_{m'h'} = 1 - \pi_{mh}.$$

Note that both π_{mh}, $\pi_{m'h'}$ are in $[0, 1]$. This implies that the solution to a household's problem is also unique for households with income in $[\widehat{y} + z^{\widehat{y}*}_{m'h'}, \widehat{y} + z^{\widehat{y}*}_{mh}]$. □

Definition 3. *There is perfect income sorting among a set of locations if the support of income distribution in any location has an empty intersection with any other location's support of income distribution.*

Theorem 1. *In equilibrium, there is never perfect income sorting among census tracts or among municipalities.*

Proof. By Proposition 3, between any two consecutive census tracts there exist households that randomizes between both tracts. This proves imperfect sorting among census tracts. Since municipalities are comprised of census tracts, there always exists at least $M - 1$ pairs of municipalities where each pair includes households with the same income levels. □

1.4 Calibration

The model is calibrated to Rhode Island data.[11] The data originally have 14 municipalities and 233 census tracts; I cluster the municipalities into 5 and the census tracts into 30.[12] In other words, I obtain a new dataset where there are 5 municipalities and 6 house types in each municipality. The resulting income

[11] Appendix A provides the reasons for working with Rhode Island.

[12] See Appendix B for details.

distributions are plotted in Figures 1.10 through 1.13. Summary statistics for these representative municipalities are provided in Table 1.3. Therefore, $M = 5$ and $H = 6$ in the computational analysis, creating a total of 30 different house types. Housing property tax rates (τ) and net state aid per household (NSA) for each municipality are taken exogenously from data.[13] Table 1.4 summarizes these numbers.

Table 1.3: Characteristics of representative municipalities

Mun.	Frac. of HH's	Med. Inc.	Med. House Val.	Per HH Pub. Spend.
I	0.1	\$39,613	\$113,250	\$3,296
II	0.15	\$34,963	\$116,300	\$4,071
III	0.15	\$44,529	\$158,766	\$6,559
IV	0.23	\$26,867	\$101,700	\$5,831
V	0.37	\$40,788	\$111,866	\$4,296

Table 1.4: Housing property tax rates and net state aid

Municipality	τ	NSA
I	2.9%	\$485
II	2.5%	\$1,674
III	2.2%	\$471
IV	3.4%	\$2,515
V	2.6%	\$1,161

[13] See Appendix A for more details on data.

The data also provide the supply of the 30 different house types. Table 1.5 shows the supply of each different house quality type in each municipality. I work with fractions instead of nominal numbers to be able to compare supply with household demand. The housing supply includes both renter-occupied and owner-occupied houses.

Table 1.5: House supply

Mun. \ House Quality	Q1	Q2	Q3	Q4	Q5	Q6
I	0.0045	0.0041	0.0193	0.0725	0.0104	0.0016
II	0.0006	0.0083	0.0201	0.1054	0.0085	0.0002
III	10^{-5}	0.0085	0.0122	0.0737	0.0373	0.0117
IV	0.0070	0.0210	0.0497	0.1316	0.0263	0.0023
V	0.0020	0.0148	0.0534	0.2461	0.0468	10^{-5}

I generate $10,000$ households with different income levels, where the income of each household is determined to replicate the observed income distribution in the data (Figure 1.31). The measure of each household type is set to 10^{-4} assuming uniform distribution.

To determine the annual rent r^*_{mh} for house type mh, I assume the real annual interest rate is 5%. Equation (1.3) then implies,

$$r(q_{mh}) = \frac{p^*_{mh}}{21}. \tag{1.17}$$

The functional form for the Bernoulli utility is:

$$U(c_{mh}, q_{mh}, E_{mh}) = q_{mh} c_{mh}^{\alpha} (\ln E_{mh})^{\gamma},$$

where $\alpha > 0$, $\gamma > 0$ and $\alpha + \gamma \leq 1$. It should be noted that this functional form is consistent with all Assumptions 1- 6.

The model has 30 house quality parameters, q_{mh}'s, and two utility parameters, α and γ, for a total of 32 parameters.[14] Let us define $\theta \equiv (\alpha, \gamma, \{\{q_{mh}\}_{m=1}^{M=5}\}_{h=1}^{H=6})$. I solve the following optimization problem to pin down these parameters:

$$\min_{\theta} \left(\int_y \frac{c_{mh}^{y*}(\theta)}{y} dF(y) - T_1^{data} \right)^2 + \left(\frac{\frac{1}{M}\sum_{m=1}^{M=5} E_m^*(\theta)}{max\{E_m^*(\theta)\}_{m=1}^{M=5}} - T_2^{data} \right)^2 + \sum_{m=1}^{M=5} \sum_{h=1}^{H=6} (I_{p_{mh}^*}(\theta) - 1)^2$$

where:

T_1^{data}: Average ratio of non-housing consumption to income in the data,

T_2^{data}: Ratio of mean public spending per household to maximum public spending per household across municipalities in the data, and

$I_{p_{mh}^*}$'s are defined as follows for each house type in any municipality:

$$I_{p_{m1}^*} = \begin{cases} 1 & \text{if } p_{m1}^* \in [\$0 - \$25,000] \\ 0 & \text{o.w.} \end{cases}$$

$$I_{p_{m2}^*} = \begin{cases} 1 & \text{if } p_{m2}^* \in [\$25,000 - \$50,000] \\ 0 & \text{o.w.} \end{cases}$$

[14]The computational algorithm used to solve the model for given parameters is explained in Appendix C.

$$I_{p^*_{m3}} = \begin{cases} 1 & \text{if } p^*_{m3} \in [\$50,000 - \$90,000] \\ 0 & \text{o.w.} \end{cases}$$

$$I_{p^*_{m4}} = \begin{cases} 1 & \text{if } p^*_{m4} \in [\$90,000 - \$175,000] \\ 0 & \text{o.w.} \end{cases}$$

$$I_{p^*_{m5}} = \begin{cases} 1 & \text{if } p^*_{m5} \in [\$175,000 - \$400,000] \\ 0 & \text{o.w.} \end{cases}$$

$$I_{p^*_{m6}} = \begin{cases} 1 & \text{if } p^*_{m6} \in [\$400,000 - \$1,000,000] \\ 0 & \text{o.w.} \end{cases}$$

In words, $I_{p_{mh}}$ is equal to 1 if the equilibrium value for house type mh lies in an interval, given by data, and 0 otherwise. In the dataset, for each house type in every municipality, I observe the number of houses and the interval in which each value lies. These value intervals are common across municipalities for each house type. For example, in the data the value of the lowest-quality house lies in $[\$0 - \$25,000]$ in every municipality. In this case, I am targeting an equilibrium value for the lowest-quality house types in every municipality such that each value belongs to $[\$0 - \$25,000]$.

The level of public spending per household is observable from the data for each municipality. The mean across municipalities is equal to $\$4,810$ per household, and maximum public spending per household is $\$6,559$. Therefore, T_2^{data} is equal to 0.73. I also need to determine T_1^{data}. This statistic is not directly observable from the data. I determine it using relations from the model as follows. Consider the household with income y. This household will choose either house type mh or $m'h'$ for sure or randomizes between alternatives mh and $m'h'$ in equilibrium. The budget constraints normalized by income for this household in both states are:

$$\frac{c_{mh}^{y*}}{y} + \frac{r_{mh}^{*}}{y} + \frac{\tau_m p_{mh}^{*}}{y} = 1 + \frac{z_{mh}^{y*}}{y}$$
$$\frac{c_{m'h'}^{y*}}{y} + \frac{r_{m'h'}^{*}}{y} + \frac{\tau_{m'} p_{m'h'}^{*}}{y} = 1 + \frac{z_{m'h'}^{y*}}{y}.$$

These constraints imply that:

$$\pi_{mh}^{y*}\frac{c_{mh}^{y*}}{y} + \pi_{m'h'}^{y*}\frac{c_{m'h'}^{y*}}{y} = \pi_{mh}^{y*}\left(1 + \frac{z_{mh}^{y*}}{y} - \frac{r_{mh}^{*}}{y} - \frac{\tau_m p_{mh}^{*}}{y}\right) + \pi_{m'h'}^{y*}\left(1 + \frac{z_{m'h'}^{y*}}{y} - \frac{r_{m'h'}^{*}}{y} - \frac{\tau_{m'} p_{m'h'}^{*}}{y}\right).$$

Using (1.1) and (1.17) and defining $\zeta^y \equiv \pi_{mh}^{y*}\frac{c_{mh}^{y*}}{y} + \pi_{m'h'}^{y*}\frac{c_{m'h'}^{y*}}{y}$ yields:

$$\zeta^y = 1 - \left(\pi_{mh}^{y*}\frac{r_{mh}^{*}}{y} + \pi_{m'h'}^{y*}\frac{r_{m'h'}^{*}}{y}\right) - 21\left(\pi_{mh}^{y*}\frac{\tau_m r_{mh}^{*}}{y} + \pi_{m'h'}^{y*}\frac{\tau_{m'} r_{m'h'}^{*}}{y}\right).$$

Now I can find the average of ζ across households as:

$$\int_y \zeta^y dF(y) = 1 - \overline{\left(\frac{r}{y}\right)} - 21\tau\overline{\left(\frac{r}{y}\right)}, \tag{1.18}$$

where I assume tax rates are the same across municipalities. $\overline{\left(\frac{r}{y}\right)}$ stands for the average rent share in income. Following Davis and Ortalo-Magne (2011), I set $\overline{\left(\frac{r}{y}\right)} = 0.18$, which includes only the contract rent as a share of income. Moreover, I set $\tau = 0.016$, which is the average residential property tax rate in Rhode Island given by Emrath (2002). Computing the right hand side of (1.18) at these values implies $\int_y \zeta^y dF(y) = 0.7568$, which is the estimate of T_1^{data}. Tables 1.6 through 1.8 provide the fit of the calibration.

Table 1.6: Calibration: Utility parameters and quality parameters for municipalities I and II

Parameter	Value	Target	Data	Model
α	0.6	Mean Cons. Exp. Sh.	0.75	0.79
γ	0.2	$\frac{mean(E)}{max(E)}$	0.73	0.76
Mun. I				
q_{11}	4.2861	House Value Int.	\$0-\$25,000	\$36,221
q_{12}	5.0311	House Value Int.	\$25,000-\$50,000	\$39,767
q_{13}	5.4378	House Value Int.	\$50,000-\$90,000	\$48,300
q_{14}	5.8580	House Value Int.	\$90,000-\$175,000	\$75,374
q_{15}	6.3651	House Value Int.	\$175,000-\$400,000	\$204,760
q_{16}	7.1512	House Value Int.	\$400,000-\$1,000,000	\$658,280
Mun. II				
q_{21}	4.2493	House Value Int.	\$0-\$25,000	\$37,982
q_{22}	4.9905	House Value Int.	\$25,000-\$50,000	\$41,446
q_{23}	5.3951	House Value Int.	\$50,000-\$90,000	\$49,797
q_{24}	5.8131	House Value Int.	\$90,000-\$175,000	\$75,256
q_{25}	6.3177	House Value Int.	\$175,000-\$400,000	\$208,170
q_{26}	7.0997	House Value Int.	\$400,000-\$1,000,000	\$665,420

1.5 Results

This section presents the calibrated model's performance with respect to empirical facts mentioned in the introduction. It is worth noting that none of these empirical facts are targeted in the calibration.

It should be noted that my model implies that exante identical income households end up at different income levels expost because of the gambling prize received.[15] Since gambling income is not observed in the data, what I refer to as

[15]Equilibrium inequality caused by lotteries among identical income households is analyzed in Rosen (1997) and Rosen (2002).

Table 1.7: Calibration: Quality parameters for municipalities III and IV

Parameter	Value	Target	Data	Model
Mun. III				
q_{31}	4.3696	House Value Int.	\$0-\$25,000	\$40,698
q_{32}	5.1025	House Value Int.	\$25,000-\$50,000	\$46,712
q_{33}	5.5027	House Value Int.	\$50,000-\$90,000	\$60,941
q_{34}	5.9161	House Value Int.	\$90,000-\$175,000	\$117,800
q_{35}	6.4150	House Value Int.	\$175,000-\$400,000	\$279,200
q_{36}	7.1883	House Value Int.	\$400,000-\$1,000,000	\$987,880
Mun. IV				
q_{41}	4.2077	House Value Int.	\$0-\$25,000	\$33,725
q_{42}	4.9377	House Value Int.	\$25,000-\$50,000	\$36,658
q_{43}	5.3389	House Value Int.	\$50,000-\$90,000	\$43,702
q_{44}	5.7534	House Value Int.	\$90,000-\$175,000	\$64,809
q_{45}	6.2537	House Value Int.	\$175,000-\$400,000	\$180,610
q_{46}	7.0291	House Value Int.	\$400,000-\$1,000,000	\$574,760

Table 1.8: Calibration: Quality parameters for municipality V

Parameter	Value	Target	Data	Model
Mun. V				
q_{51}	4.2263	House Value Int.	\$0-\$25,000	\$37,463
q_{52}	4.9665	House Value Int.	\$25,000-\$50,000	\$40,543
q_{53}	5.3707	House Value Int.	\$50,000-\$90,000	\$47,752
q_{54}	5.7882	House Value Int.	\$90,000-\$175,000	\$68,901
q_{55}	6.2920	House Value Int.	\$175,000-\$400,000	\$192,450
q_{56}	7.0730	House Value Int.	\$400,000-\$1,000,000	\$635,750

"income" in all results is simply exante income. Figure 1.32 compares the data and the model with respect to the relation between the median income of a municipality and the percentage of the municipal household population with an annual income above $\$40,000$. The model-implied correlation is 0.90, whereas the data analog is

0.97. Figures 1.33 through 1.37 compare the model-implied income distribution with data for each municipality. These figures suggest that the model is more successful in terms of matching the tails of income distribution. Among all municipalities, the model's fit for municipality IV seems the best. As reported in Table A.1, municipality IV corresponds to the city of Providence (largest city in Rhode Island) in the data. Moreover, the model-implied ratio of the within-municipality variance of income to the statewide variance of income is 0.68. The corresponding number in the data is 0.94.

Increasing the number of house types in each municipality is expected to improve the fit of model. For instance consider the richest municipality (III) and the poorest municipality (IV). According to Figures 1.33 through 1.37, middle-income households do not live in municipality III and high-income households do not live in municipality IV. If there were more house types in each municipality, then middle-income households would also be able to find a suitable house for themselves in municipality III and high-income households would be able to find a suitable house in municipality IV. Figure 1.38 compares the data and the model with respect to the relation between the median income of a census tract and the percentage of the census tract's household population with an annual income above $40,000. The model implied correlation is 0.53 whereas the data analog is 0.90. Figure 1.39 compares model and data with respect to the percentage of census tracts a particular income group lives. As the figure shows, the poorest households (with income between [$\$0 - \$10,000$]) live in approximately 50% of the census tracts whereas the richest households (with income greater than $\$200,000$) live in only 20% of the census tracts. The corresponding numbers in the data are 100% and 80%, respectively. Thus, the model is consistent with imperfect income sorting across census tracts; however, there is room for further improvement.

As Figure 1.9 shows, households with the same rent share of income but different incomes reside in the same municipalities. Figures 1.40 through 1.44 compare the model-implied distribution of households grouped with respect to their rent share of income across municipalities with the distribution in the data. The fit of the model worsens as the rent share of income increases. It is worth reminding that 46% of the population belongs to the first rent share of income group.

The intuition for the success of the model with respect to this fact can be explained as follows. Consider two house types in the same municipality. Also consider two households with income levels y and $2y$, and assume the income y household assigns positive probability to house type 1 and the income $2y$ household assigns positive probability to house type 2. Also assume the respective values of these houses are p and $2p$. Then the rent share of income for the household with income y living in house type 1 is p/y, and the rent share in income for the household with income $2y$ living in house type 2 is also p/y. Given that these houses are in the same municipality, this shows that two different households with the same rent share in income may choose to live in the same municipality in the model.

As suggested in Figures 1.40 through 1.44, some household types belonging to a particular income and rent share of income group in the data do not exist in my model. For instance consider Figure 1.44. In the data, there exist households with income between $\$75,000$ and $\$100,000$ that spend more than 29% of their income for rent. This particular group is missing in my model. It is expected that this inconsistency between the model and the data could be resolved if there were more house types in each municipality. Table 1.9 compares the model-implied correlations with those in the data. The median income and median house value correlation and then median house value and per household public spending correlation in the data are successfully predicted by the model.

Moreover, in the data the median income and Gini coefficient of income correlation is around -0.59. The model correctly predicts approximately 65% of the correlation observed in the data. The intuition for matching this fact may be thought as follows. In the model, there is a positive correlation between the median incomes and median house values in each municipality. Therefore it is more expensive to live in richer municipalities. This zoning effect eliminates poor people from living in rich municipalities whereas rich people can choose to live in poor municipalities because of the imperfect sorting effect. Therefore, the distribution of income in a poor municipality will have a higher variance compared with living in a richer municipality.

Table 1.9: Other facts

Correlations (Not Targeted in Estimation)	Data	Model
Median Income & Median House Value	0.7250	0.7611
Median House Value & Per Household Public Spending	0.5571	0.5687
Median Income & Gini Index of Income	-0.5993	-0.3754

1.6 Counterfactual policy experiments

With the calibrated model, which is consistent with the empirical facts, at hand, two policy questions are answered in this section: 1) How much does property tax competition affect the sorting of households across municipalities? 2) How much would income sorting change if the supply of each house type were equal across municipalities?

1.6.1 Uniform tax policy

Empirical studies such as the one by Brueckner and Saavedra (2001) find evidence of property tax competition between municipalities. In several papers, such as those by Wilson (1999), Brueckner (2000), and Brueckner (2004), it is argued that property tax competition has two opposing effects in the economy. The first effect is under provision of local public good since competition reduces tax rates. The second effect, following Tiebout (1956)[16], is increased efficiency in providing local public good as a result of increased household sorting.

[16] Validity of this hypothesis is subjected to several econometric tests with the most recent studies by Rhode and Strumpf (2003) and Banzhaf and Walsh (2008).

In this experiment, I question whether the property tax competition is associated with higher household sorting, the second effect discussed above. For this goal, I exogenously eliminate tax rate differentials by imposing the benchmark mean tax rate as the new tax rate in each municipality. The mean tax rate in the benchmark model (see Table 1.4) is around 2.7%. Therefore, this is the new tax rate in each municipality under the experiment. If property tax competition increases sorting in the economy, then household sorting should be lower under this experiment since tax competition is exogenously eliminated. The results reported below conclude that household sorting increases by 17% under the experiment, which is the opposite of what is expected. The intuition for this result is explained below in more detail.

The resulting income distributions in each municipality are demonstrated in Figures 1.45 through 1.49. The comparison of median incomes, per household public spending levels, and median house values for each municipality under the benchmark and experiment is provided in Table 1.10. Before commenting on these results, it should be noted that the benchmark tax rates are higher than mean tax rate in municipalities I and IV and lower than mean for municipalities II, III and V. According to Table 1.10, below-the-mean municipalities, II, III and V, experience increases in median incomes, per household public spending levels, and median house values whereas the reverse happens in above-the-mean municipalities I and IV. This finding reveals that either rich households move from municipalities I and IV to municipalities II, III and V or poor households move from II, III and V to

I and IV. Considering this together with Figures 1.45 through 1.49 reveals that rich households actually move from I and IV to II and III and at the same time poor households move from V to I and IV. The intuitive explanation for this result is as follows. A closer look at Tables 1.4 and 1.10 reveals that richer municipalities have lower residential property tax rates. Therefore, under the experiment richer municipalities experience an increase in tax rates, whereas poorer municipalities experience a decrease. But at the same time, richer municipalities experience an increase in per household public spending levels and poorer municipalities experience a decrease in per household public spending levels. Depending on the elasticity of substitution between public spending and consumption, some rich households choose to move to richer municipalities and poorer households find it more optimal to go to poorer municipalities. This, in turn, increases housing prices in rich municipalities and decreases housing prices in poor municipalities, which further increases the level of public spending. As a result, richer households agglomerate in richer municipalities and vice versa for poor households. Consequently, this policy increases the degree of income sorting in the society.

I use the following measure of sorting to provide a precise number for the increase in income sorting:

$$\Omega = \frac{\sum_{m=1}^{M} \lambda_m \ln \frac{\overline{y}_m}{\overline{y}}}{\Sigma_{m=1}^{M} \lambda_m \int_y N_m \frac{y}{\overline{y}_m} \ln \frac{y}{\overline{y}_m} f_m(y) dy} = \frac{BG}{WG},$$

where:

- λ_m is the income share of municipality m,

Table 1.10: Comparison of equilibrium values under benchmark and uniform tax experiment

Mun.	Med. Income		Public Spending Per HH		Med. House Value	
	Bench.	Exp.	Bench.	Exp.	Bench.	Exp.
I	\$62,394	\$50,277	\$3,084	\$2,783	\$74,983	\$73,773
II	\$51,658	\$59,931	\$3,693	\$4,191	\$72,893	\$80,272
III	\$78,215	\$79,637	\$5,351	\$7,225	\$157,000	\$176,130
IV	\$39,185	\$20,552	\$5,131	\$4,526	\$63,734	\$64,939
V	\$25,596	\$35,510	\$3,305	\$3,425	\$72,686	\$74,364

- $\overline{y}_m$ is the mean income in municipality m,
- $\overline{y}$ is the mean income in the society, and
- N_m is the measure of households living in municipality m.

The numerator of Ω, BG, is the between-municipality variance in income and the denominator, WG, is the within-group variance in income. The sum of the numerator and the denominator is the Theil Index. This measure is used also by Kremer and Maskin (1996) and Davidoff (2005) to measure sorting. Higher levels of Ω are associated with higher levels of sorting. Table 1.11 reports the values of WG, BG and Ω for both the benchmark and the experiment. As seen, sorting in the society increases by 17% under the experiment compared with the benchmark. Therefore, tax rate heterogeneity plays an integrating role in society.

Table 1.11: Income sorting under benchmark and uniform tax experiment

	WG	BG	Ω
Benchmark	0.5324	0.2658	0.4992
Experiment	0.5033	0.2952	0.5865

1.6.2 Mixed-income housing policy

U.S. housing policy focuses on deconcentrating poverty by subsidizing the housing consumption of poor households if they live in rich neighborhoods. The past two decades have shown that this policy had little effect on increasing integration. Motivated by this finding the U.S. government has turned to policies that not only give poor households incentives to live in rich neighborhoods, but also gives rich households incentives to live in poor neighborhoods. As noted in Schwartz and Tajbakhsh (1997), there has been little quantitative analysis on the effects of this new policy. In this section, I analyze this mixed income housing policy using the theoretical model developed and calibrated above.

To analyze mixed-income housing policy, I exogenously set the mean supply of a particular house type in the whole economy as the new supply for that type in each municipality. For instance, the mean supply of the first house type in the whole economy is 0.0028 (see Table 1.5). Under the experiment, I set the supply of the first house type as 0.0028 in each municipality. As a result, the supply of housing becomes identical across municipalities. Table 1.12 lists the new housing supply.

Compared with the benchmark, the supply of low-quality housing is increased in the rich municipality and the supply of high-quality housing is increased in the poor municipality. Moreover, the population of each municipality becomes identical under the experiment since each household in the model consumes one unit of housing. The new population of each municipality is now 0.2. The benchmark population is provided in Table 1.3. Comparison of the new population with the benchmark implies that population is increased in the first three municipalities and decreased in the last two.

Table 1.12: New house supply

Quality / Mun.	Q1	Q2	Q3	Q4	Q5	Q6
I	0.0028	0.0113	0.0309	0.1259	0.0259	0.0032
II	0.0028	0.0113	0.0309	0.1259	0.0259	0.0032
III	0.0028	0.0113	0.0309	0.1259	0.0259	0.0032
IV	0.0028	0.0113	0.0309	0.1259	0.0259	0.0032
V	0.0028	0.0113	0.0309	0.1259	0.0259	0.0032

Figures 1.50 through 1.54 show the comparison of the new income distributions with benchmark distributions for each municipality. These figures reveal that the proportion of households with income below $40,000 has increased in nearly all municipalities. Therefore, as expected, the rich population moves into poorer municipalities and the poor population moves into richer municipalities. As seen

from Table 1.13, the median income decreases in each municipality, whereas public spending per household and the median house value decrease in the richest municipality (III) and increase in others. The median income decrease is expected in rich municipalities but not in poor municipalities. A closer look at Figures 1.50 through 1.54 reveals that some middle-income households are moving from poor to rich municipalities, which causes the decrease in median incomes in poor municipalities.

Table 1.13: Comparison of equilibrium values under benchmark and mixed income housing experiment

Mun.	Med. Income		Public Spending Per Household		Med. House Value	
	Bench.	Exp.	Bench.	Exp.	Bench.	Exp.
I	\$62,394	\$53,522	\$3,084	\$3,222	\$74,983	\$84,985
II	\$51,658	\$41,299	\$3,693	\$3,952	\$72,893	\$86,910
III	\$78,215	\$69,886	\$5,351	\$4,256	\$157,000	\$115,420
IV	\$39,185	\$31,394	\$5,131	\$5,386	\$63,734	\$76,233
V	\$25,596	\$22,047	\$3,305	\$3,351	\$72,686	\$79,196

How much does a mixed-income housing policy deconcentrate poverty? To answer this question, I use the sorting metric Ω defined above. As seen in Table 1.14, Ω decreases from 0.49 to 0.22, which amounts to a 56% change. Therefore, income sorting decreases by 56% if the supply of house types is equal across municipalities.

Table 1.14: Income sorting under benchmark and mixed income housing experiment

	WG	BG	$\Omega = BG/WG$
Benchmark	0.5324	0.2658	0.4992
Experiment	0.6485	0.1436	0.2214

1.7 Conclusion

This paper proposes a testable theory that is quantitatively consistent with imperfect income sorting across municipalities and census tracts together with several correlations between municipal variables. The model setup is a simple generalization of Tiebout (1956), which predicts perfect sorting of households. More specifically, I propose a general equilibrium model in which heterogeneous households are matched with heterogeneous house types. Each municipality is heterogeneous with respect to property tax rates, local public spending per household, value and fixed stock of several house types. The value of each house type and local public spending per household are determined endogenously in equilibrium, whereas the property tax rate and stock of each house type are exogenously given. Households are heterogeneous with respect to income and derive utility from consumption, housing quality, and public spending per household. In contrast to Tiebout (1956), in my model households do not self-select themselves across census tracts and municipalities deterministically. Instead, they choose a lottery that is a vector of probabilities and payoffs for each house type in each municipality. I argue that households have an incentive to randomize across house types since their choice set is non-convex.

With the calibrated model, which is consistent with the empirical facts, at hand, two policy questions are answered: 1) How much does property tax competition affect the sorting of households across municipalities? 2) How much would income sorting change if the supply of each house type were equal across municipalities? For the first question, I conduct a counterfactual experiment that exogenously sets the mean benchmark residential property tax rate as the new tax rate in each municipality. Eliminating tax differentials causes rich households living in poor municipalities to move to rich municipalities. This increases housing prices in rich municipalities, which in turn causes poor households to relocate to poor municipalities. Therefore, income sorting increases by 17% under the first policy experiment. This finding challenges the common view that tax competition is associated with increased sorting. I argue that the previous literature ignored the general equilibrium house price effect. To answer the second question, I exogenously set the mean supply of a particular house type in the whole economy as the new supply for that type in each municipality. As a result, the supply of housing becomes identical across municipalities under the experiment. Compared with the benchmark, the supply of low-quality housing is increased in the rich municipality and the supply of high-quality housing is increased in the poor municipality. This policy is called the "mixed-income housing policy" in the paper since it creates incentives through the housing market for the poor and the rich to live in the same municipality. As a fulfillment of these intuitive expectations, income sorting in the society decreases by 56%, under this policy.

Future work aims to compare centralized vs. decentralized provision of local public education. Efficient provision of local public education favors decentralized provision, given Tiebout hypothesis, whereas equity favors centralized provision. This problem requires endogenous determination of tax rates, which is missing in my model. Moreover, location choice via lotteries can also be used in city choice models as an alternative to the spatial equilibrium concept used by Rosen (1979) and Roback (1982). As shown by Mirrlees (1972), indifference across locations based on the spatial equilibrium concept is not optimal for households. Moreover, using a lottery framework in city choice models would also be consistent with, for example, imperfect sorting of different education groups across cities. Lastly, perfect income sorting across census tracts has several implications in terms of the evolution of income inequality and economic growth over time, as shown by Benabou (1996) and Durlauf (1996). However, data support imperfect income sorting across census tracts. It may be interesting to reanalyze the interaction of household sorting with macroeconomic variables where households sort only imperfectly.

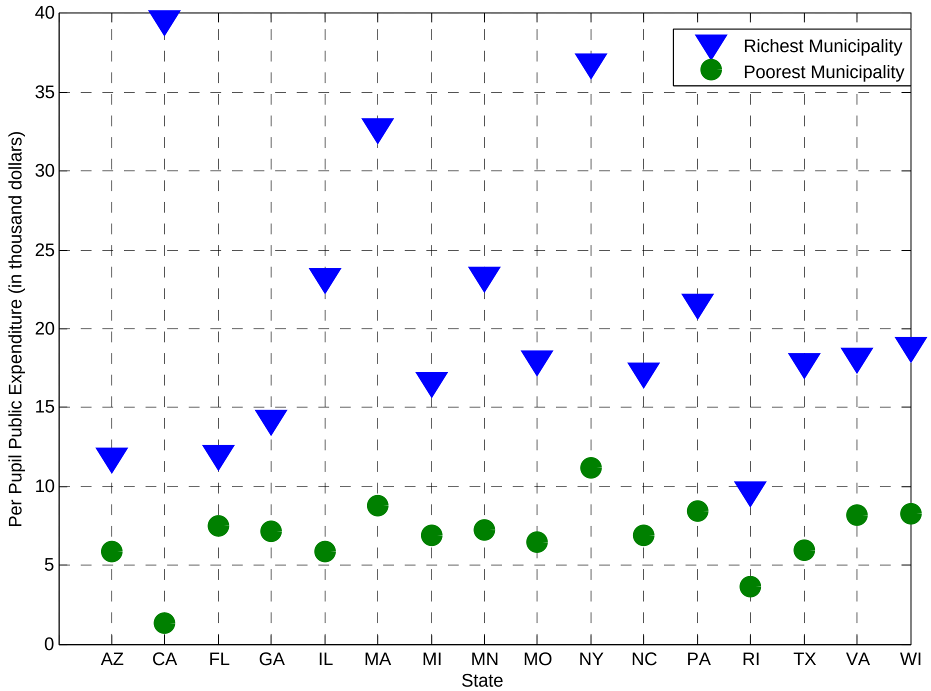

Figure 1.1: Public spending differentials across U.S. municipalities

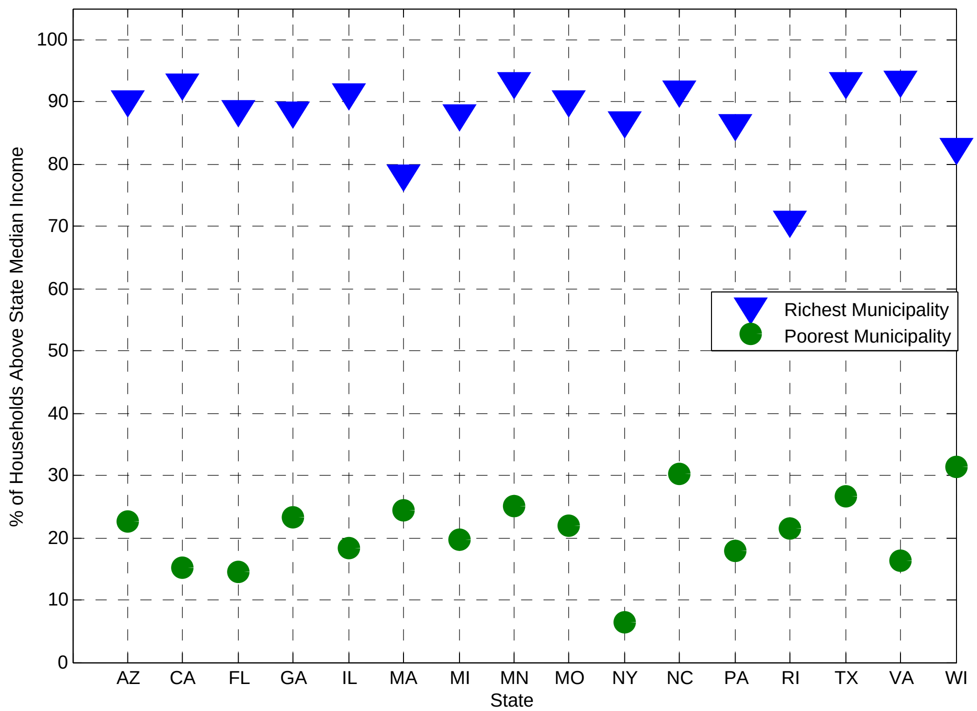

Figure 1.2: Imperfect income sorting across U.S. municipalities

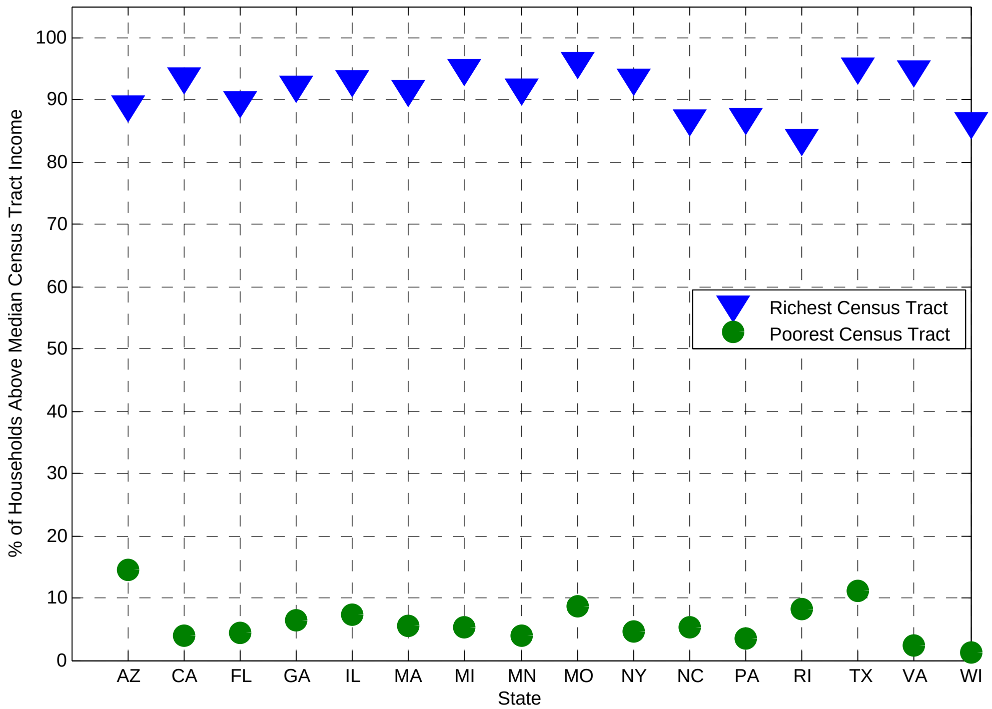

Figure 1.3: Imperfect income sorting across U.S. census tracts

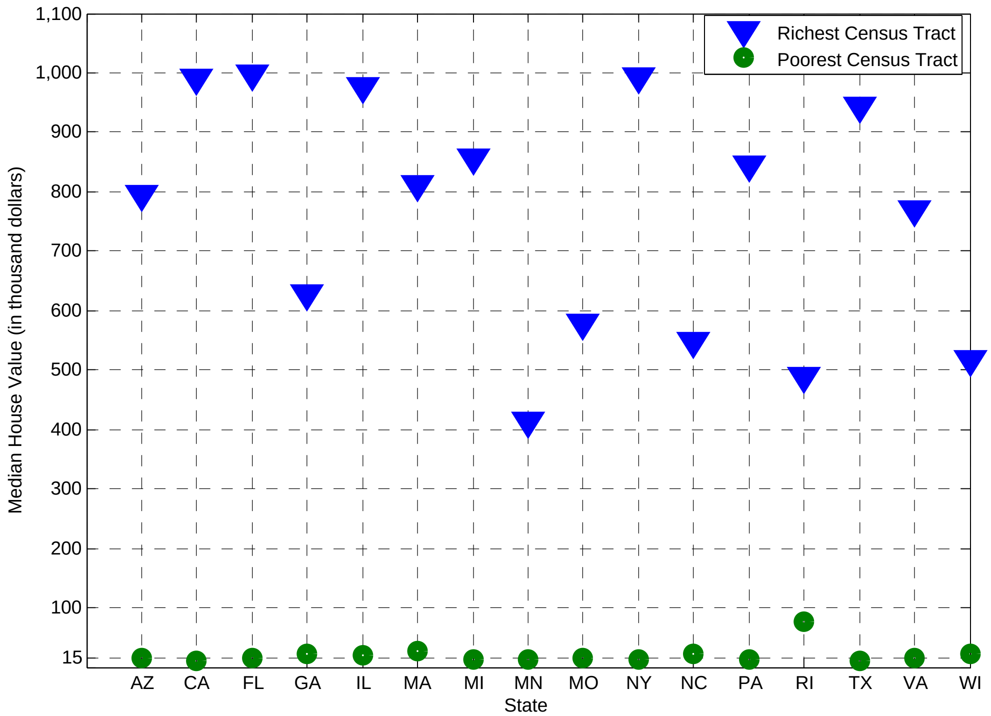

Figure 1.4: Median house value differentials across U.S. census tracts

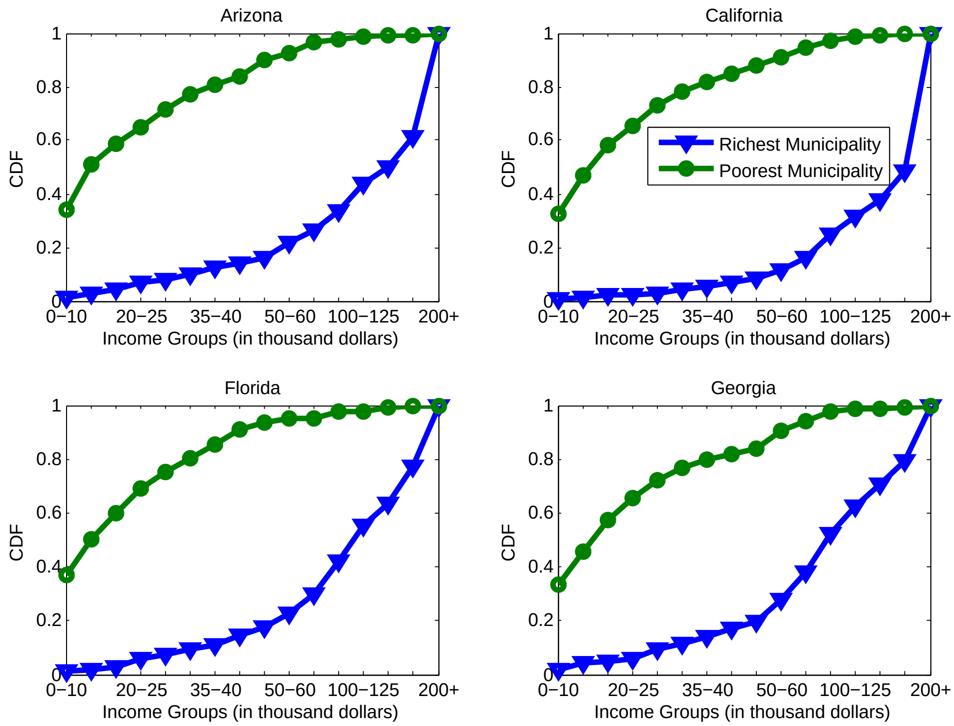

Figure 1.5: Income distribution across municipalities within Arizona, California, Florida and Georgia

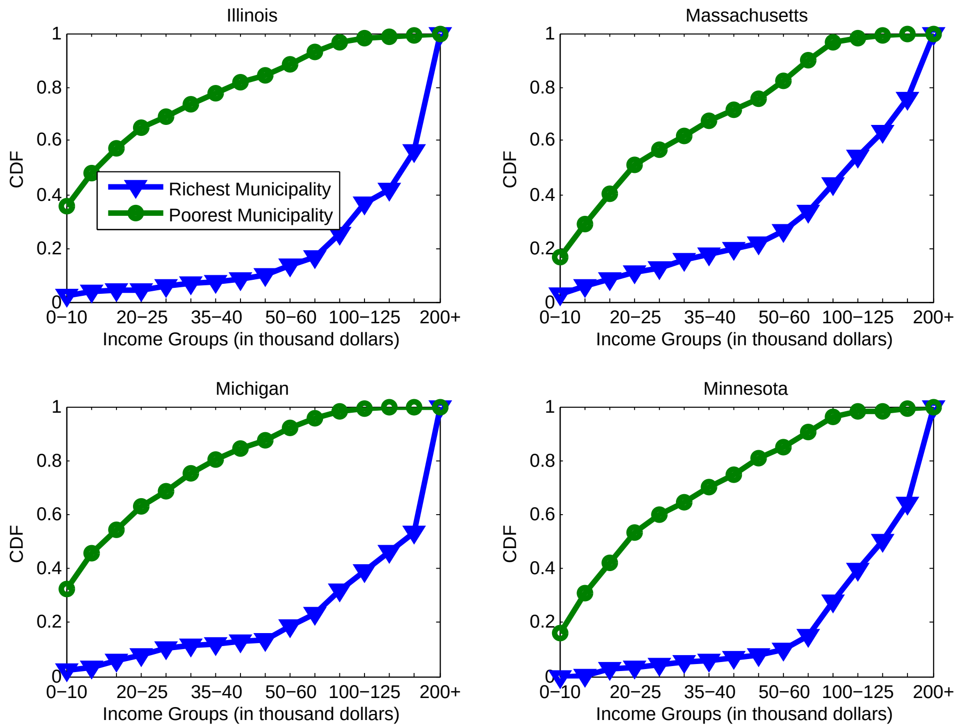

Figure 1.6: Income distribution across municipalities within Illinois, Massachusetts, Michigan and Minnesota

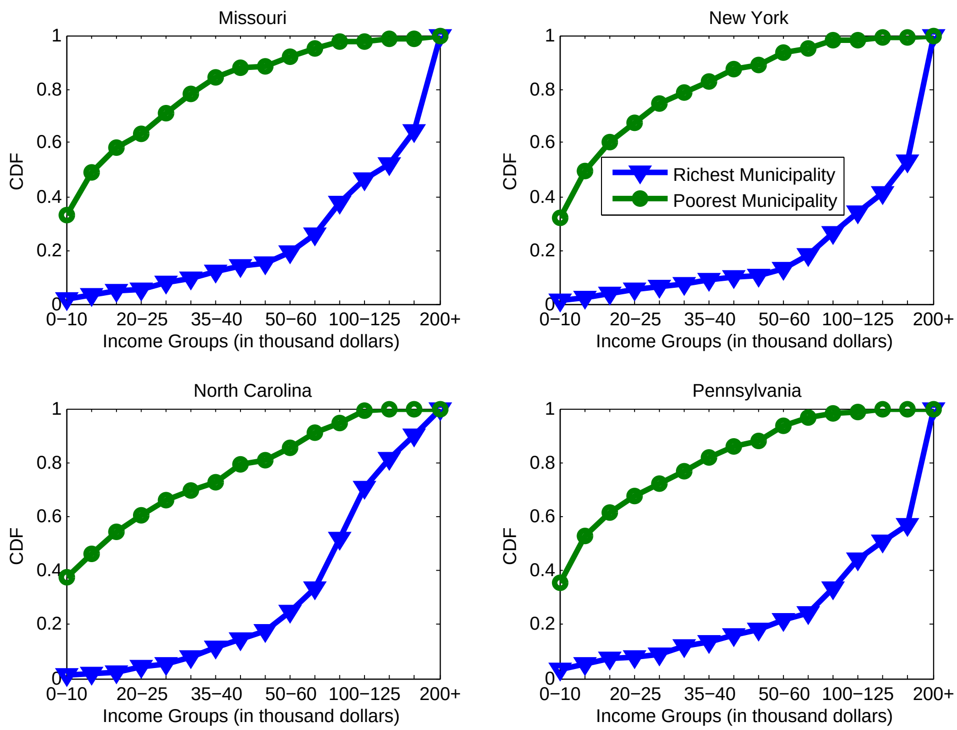

Figure 1.7: Income distribution across municipalities within Missouri, New York, North Carolina and Pennsylvania

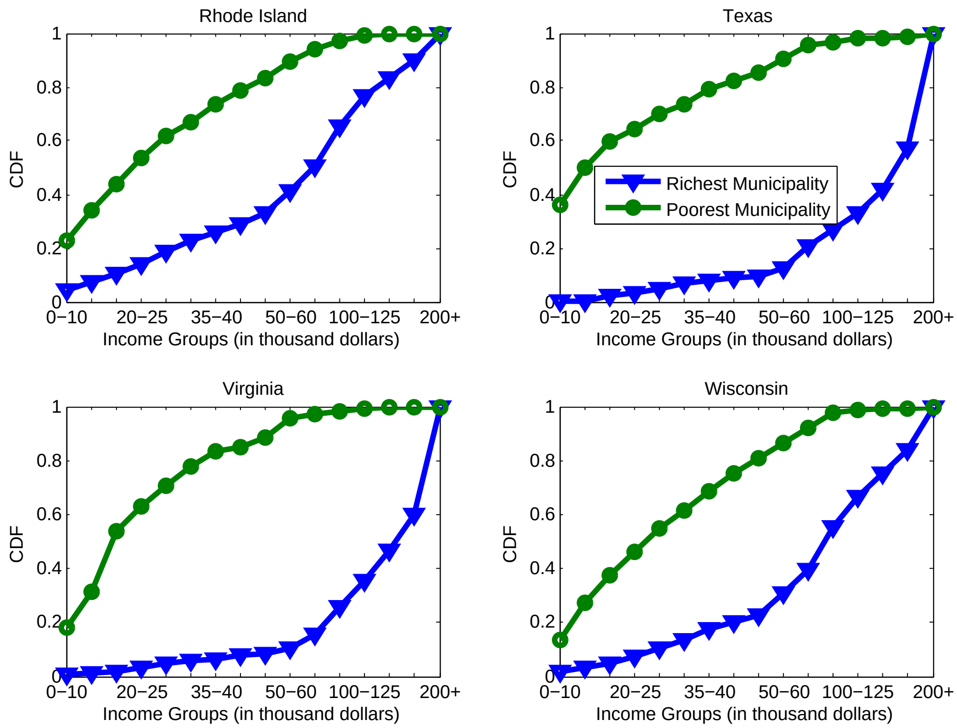

Figure 1.8: Income distribution across municipalities within Rhode Island, Texas, Virginia and Wisconsin

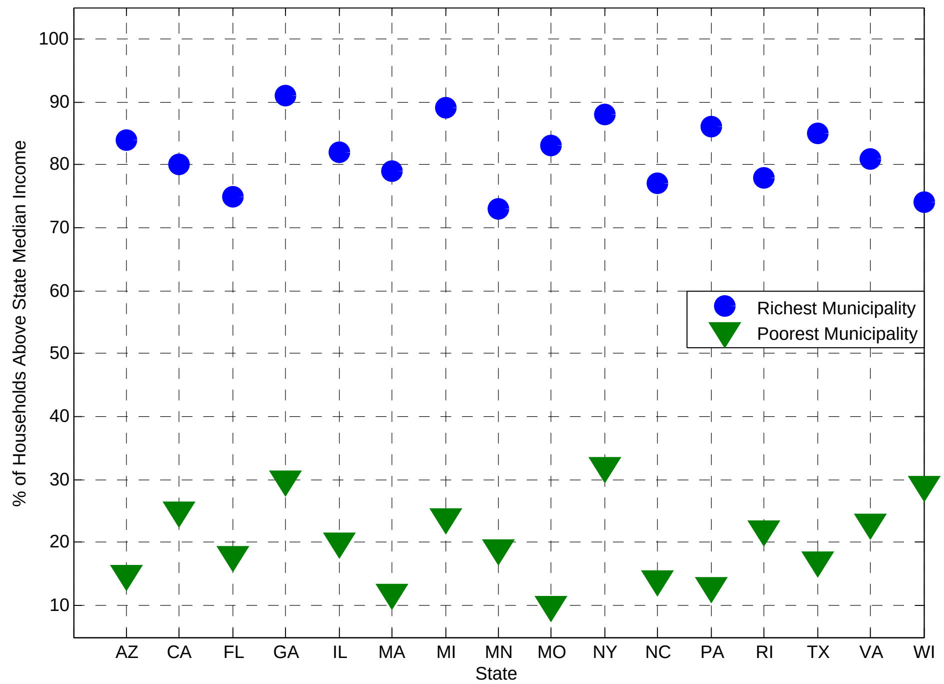

Figure 1.9: Conditional imperfect income sorting across U.S. municipalities

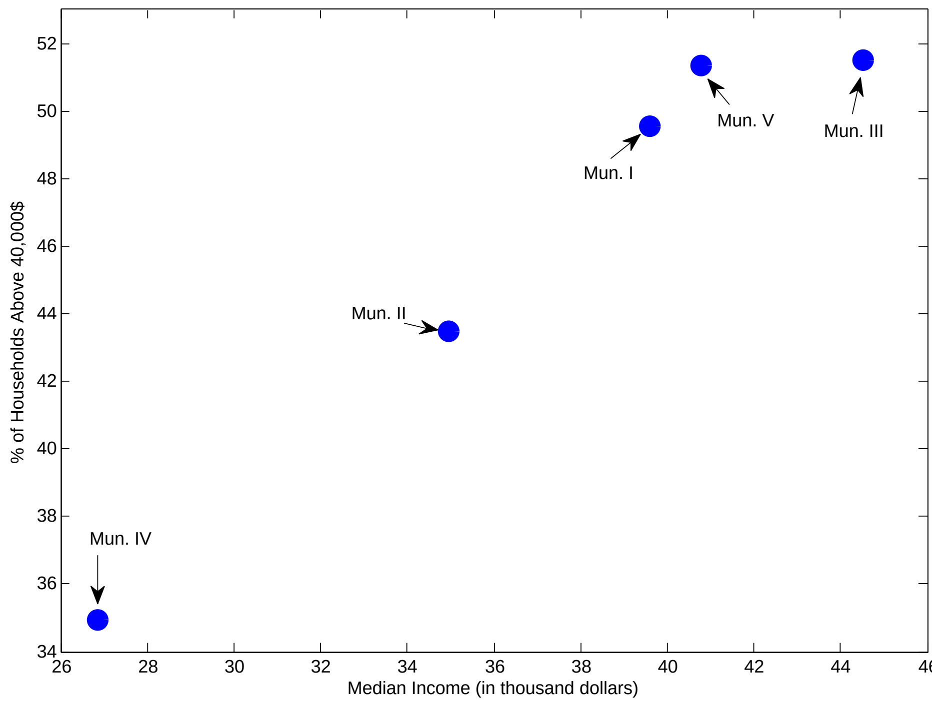

Figure 1.10: Imperfect income sorting across representative municipalities

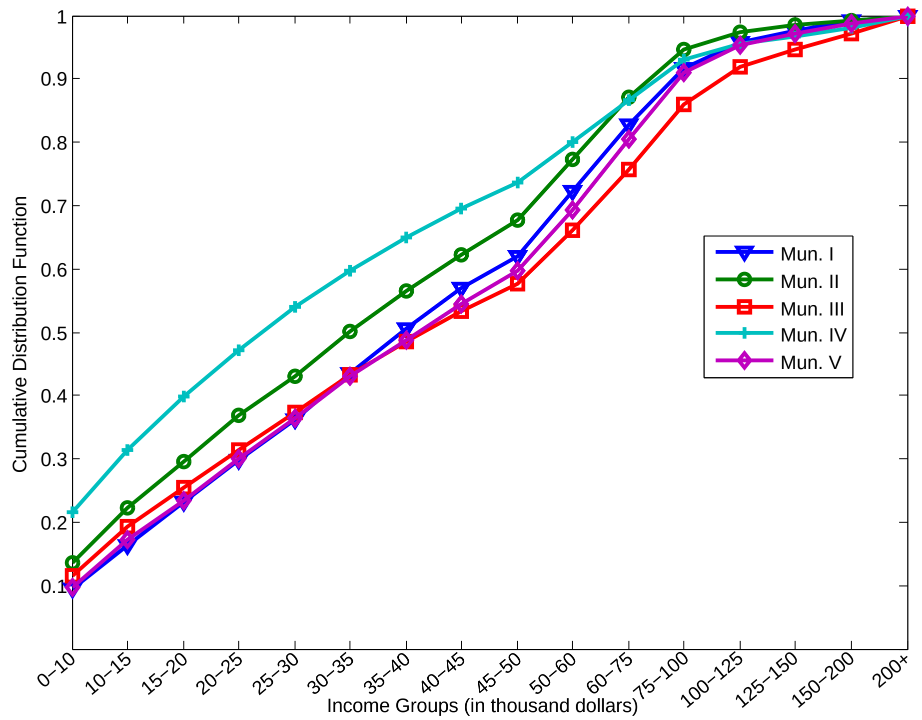

Figure 1.11: Income distribution across representative municipalities

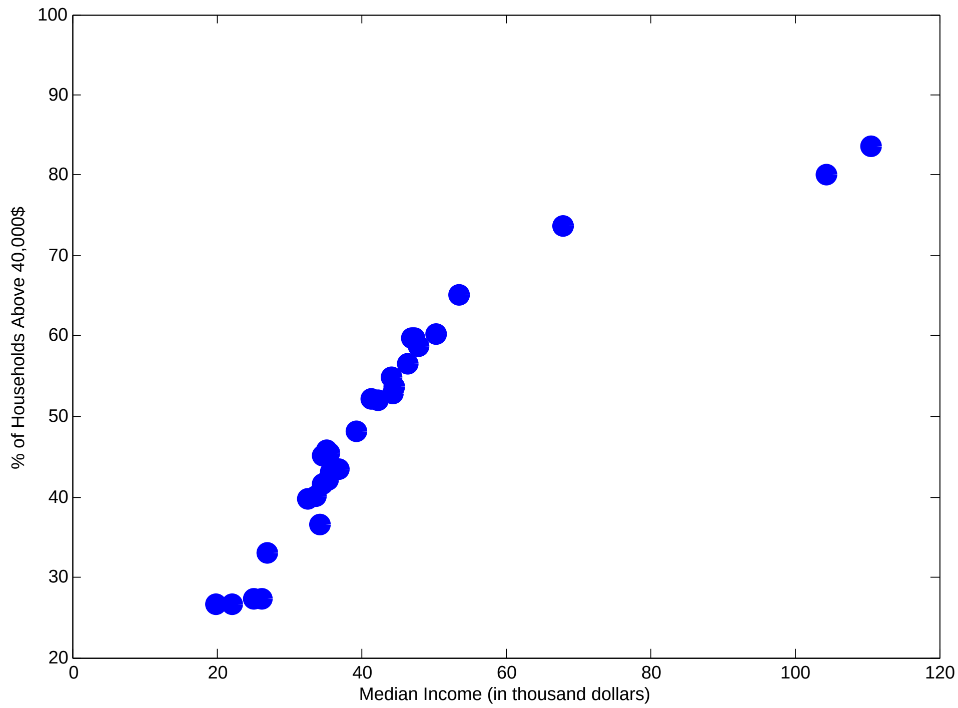

Figure 1.12: Imperfect income sorting across representative census tracts

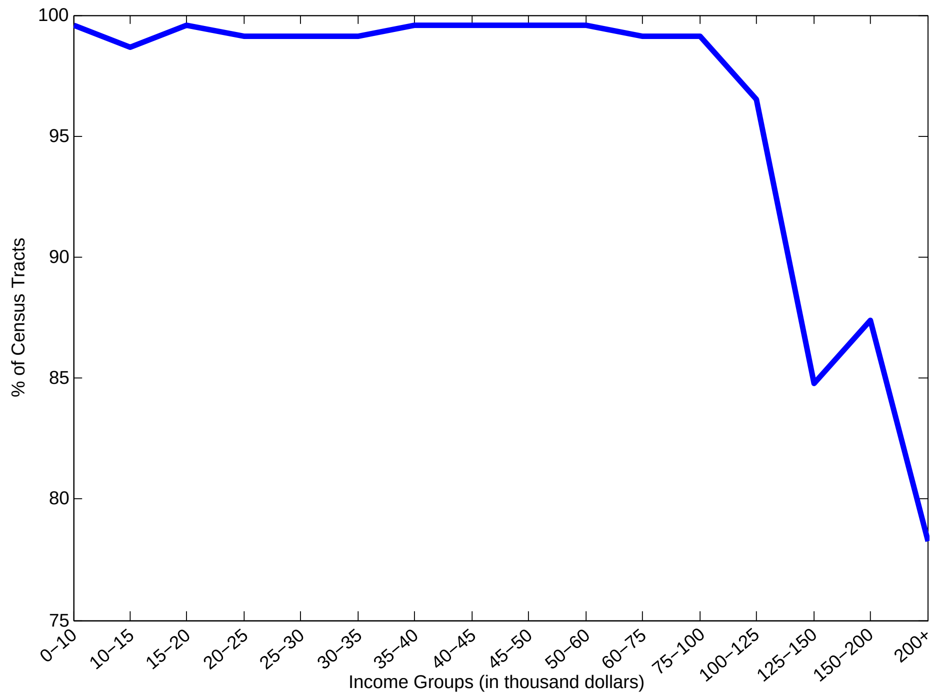

Figure 1.13: Another look at imperfect income sorting across representative census tracts

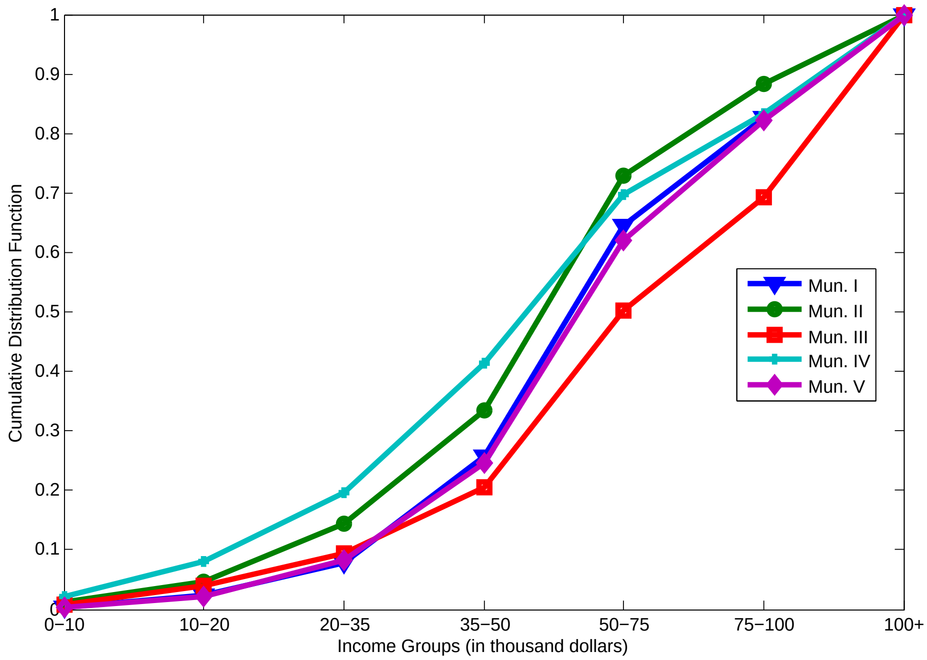

Figure 1.14: Imperfect income sorting conditional on rent share in income less than 14%

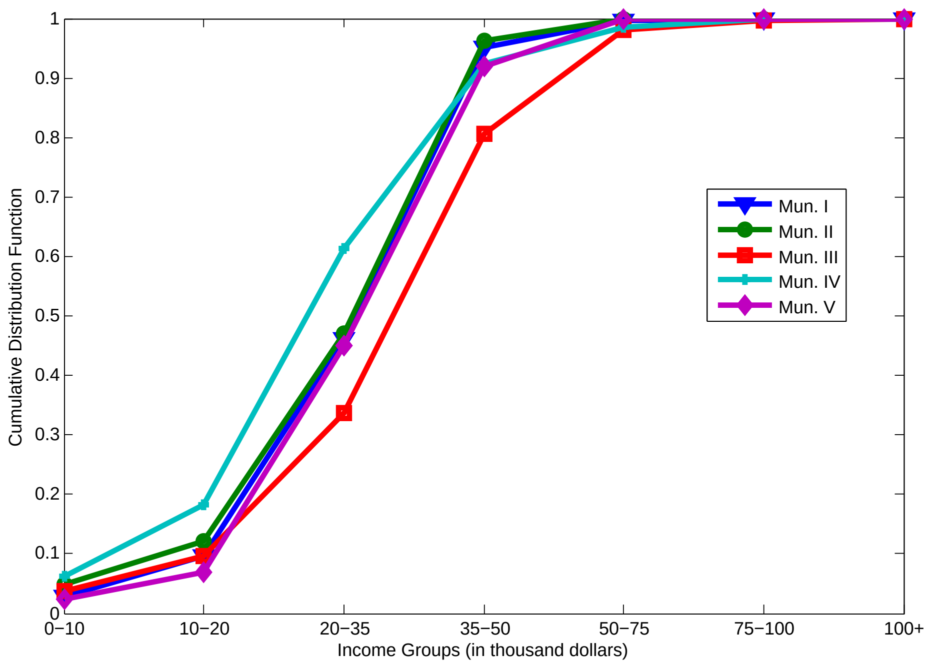

Figure 1.15: Imperfect income sorting conditional on rent share in income between 14% and 19%

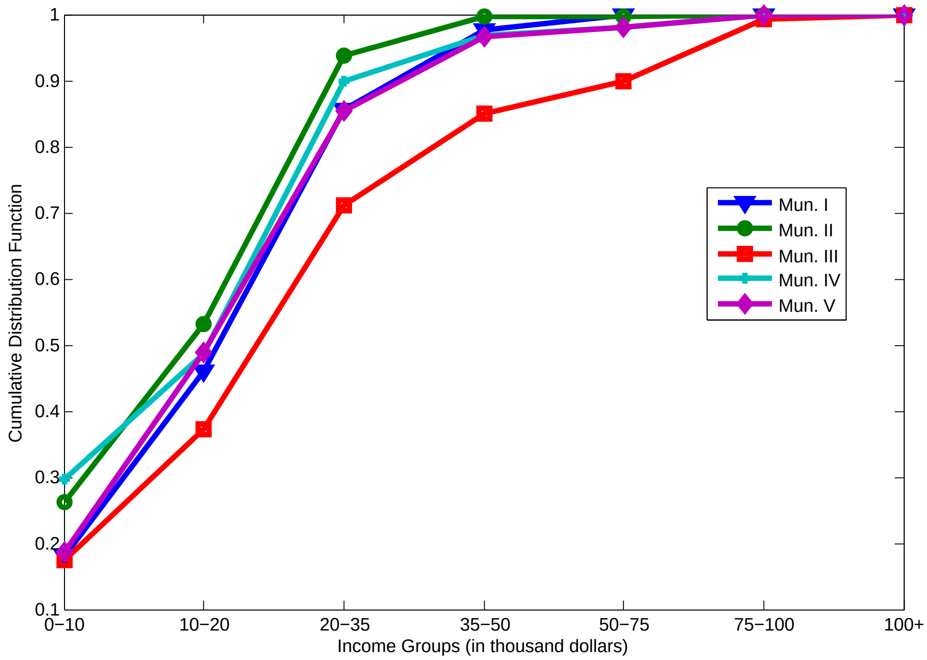

Figure 1.16: Imperfect income sorting conditional on rent share in income between 19% and 24%

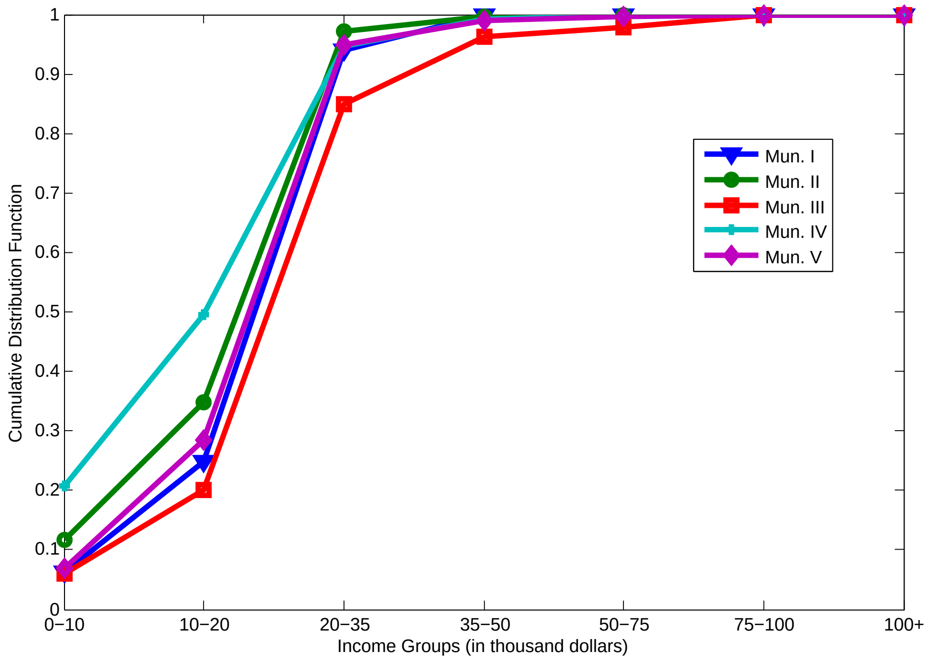

Figure 1.17: Imperfect income sorting conditional on rent share in income between 24% and 29%

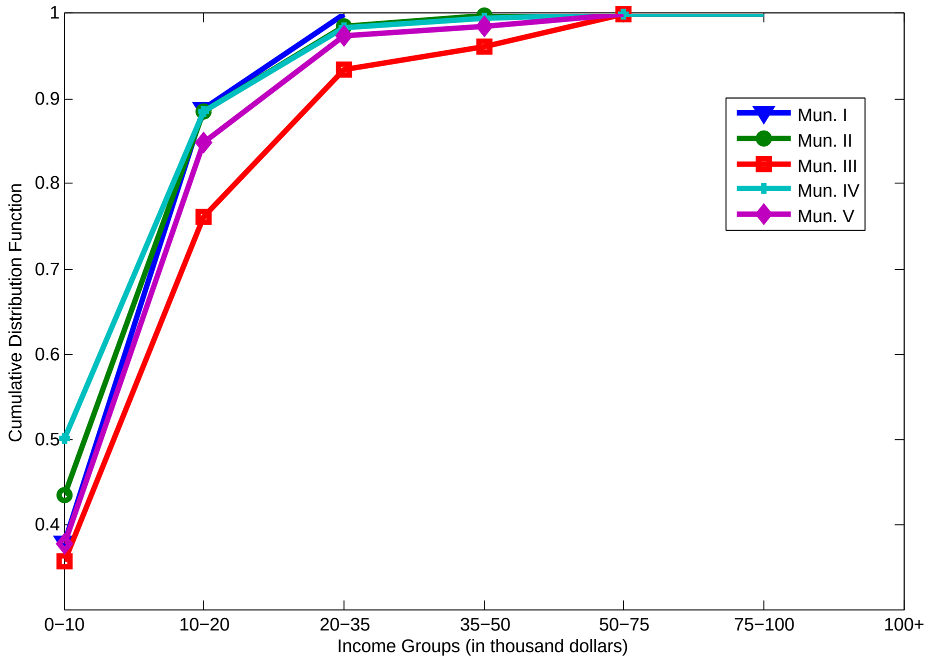

Figure 1.18: Imperfect income sorting conditional on rent share in income more than 29%

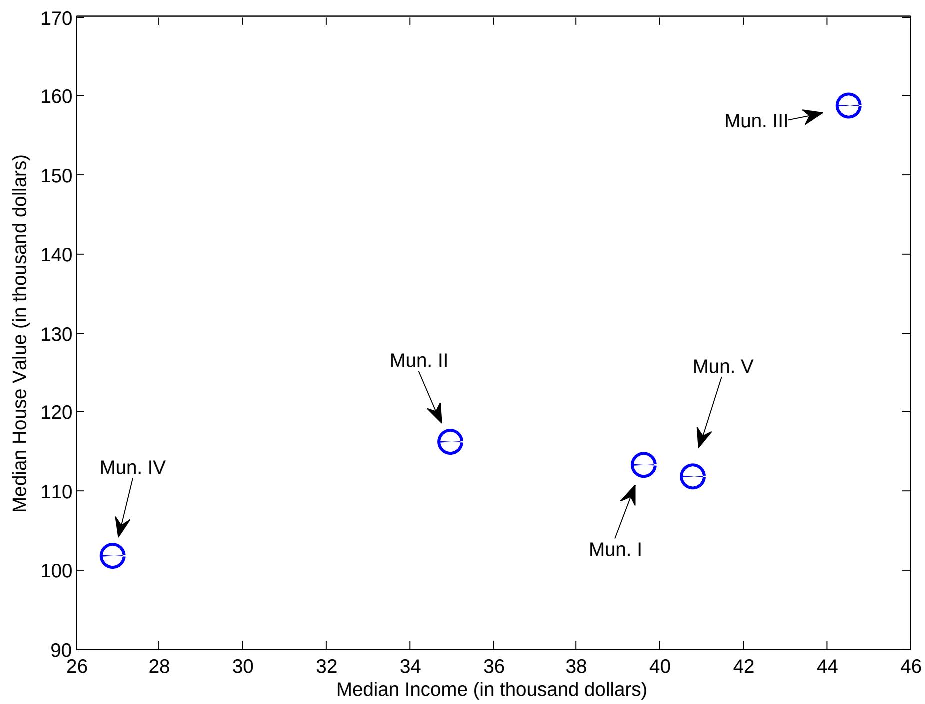

Figure 1.19: Median income vs. median house value in representative municipalities Corr=0.72

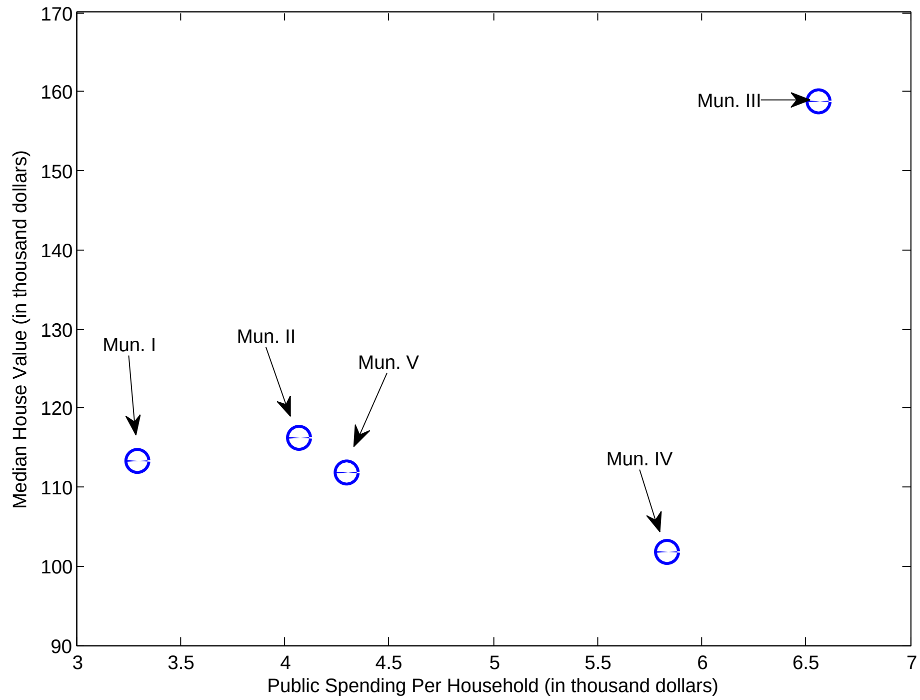

Figure 1.20: Public spending per household vs. median house value in representative municipalities Corr=0.55

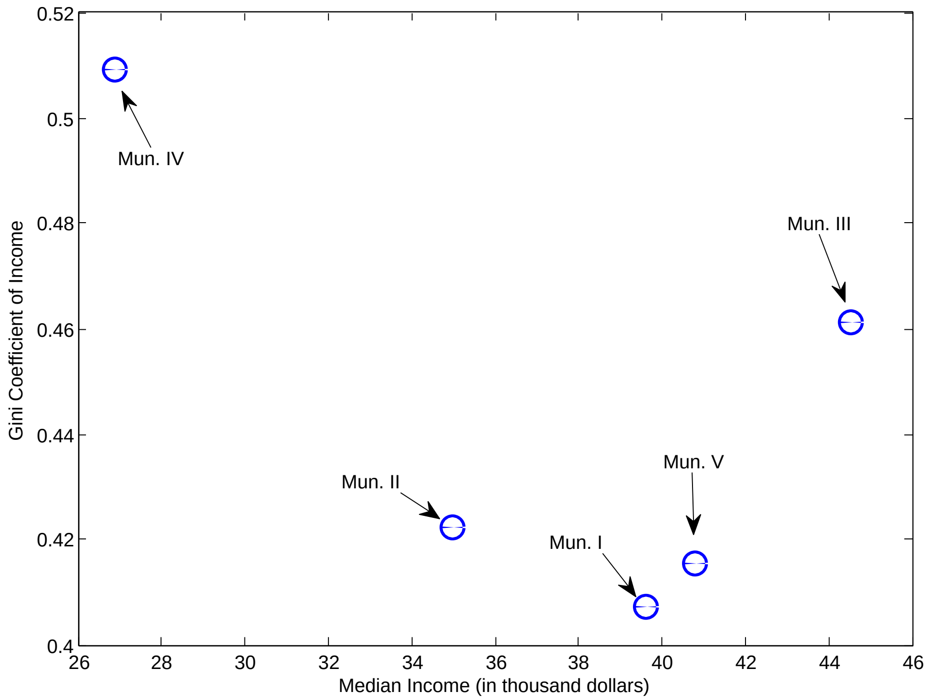

Figure 1.21: Median income vs. gini coefficient of income in representative municipalities Corr=-0.59

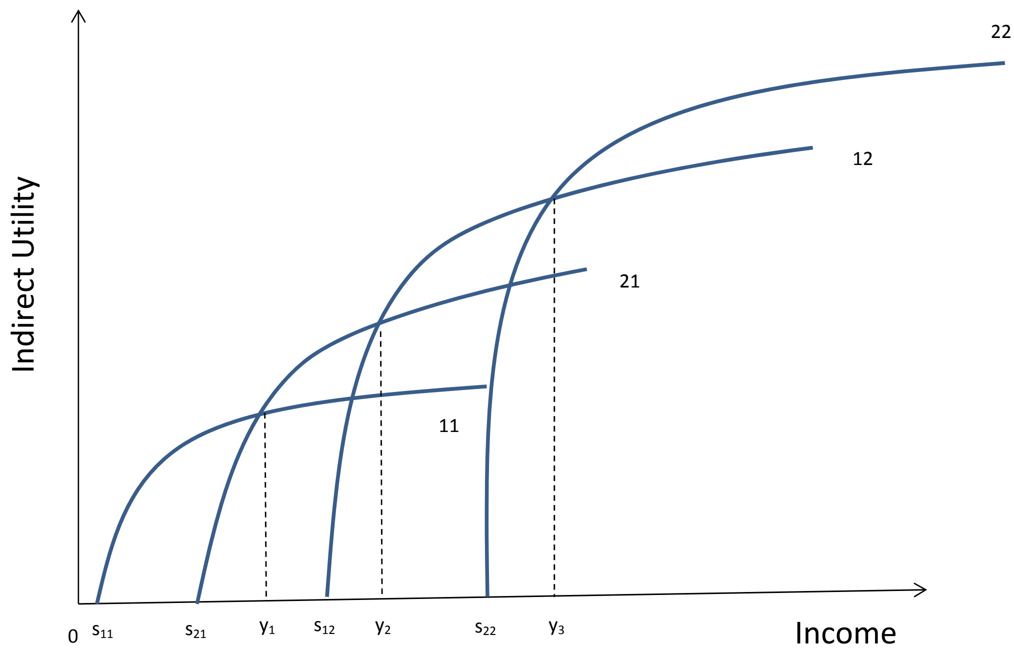

Figure 1.22: Indirect utility function for different house types

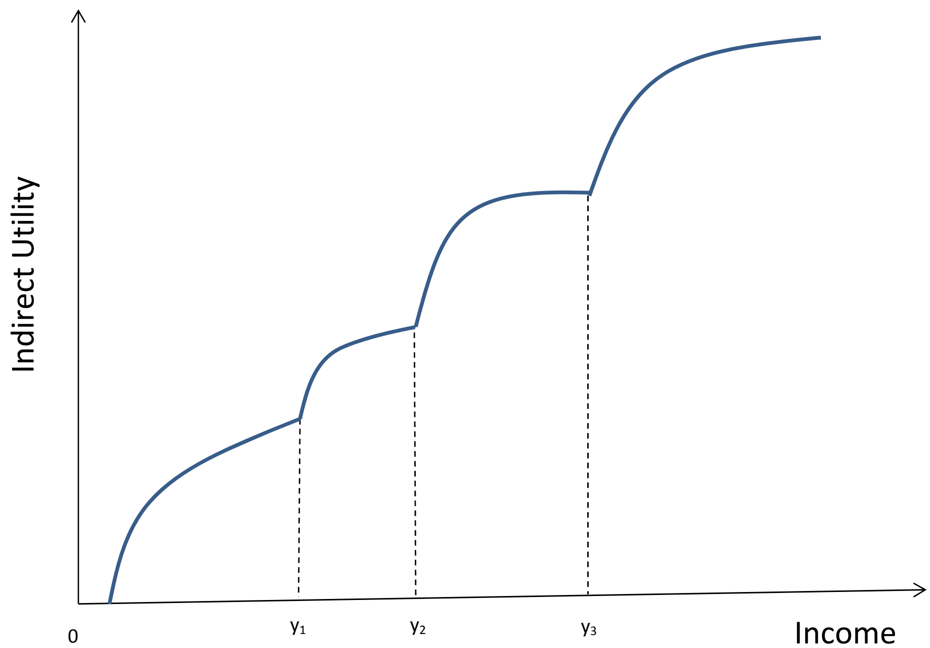

Figure 1.23: Behavior of indirect utility function under indivisibility

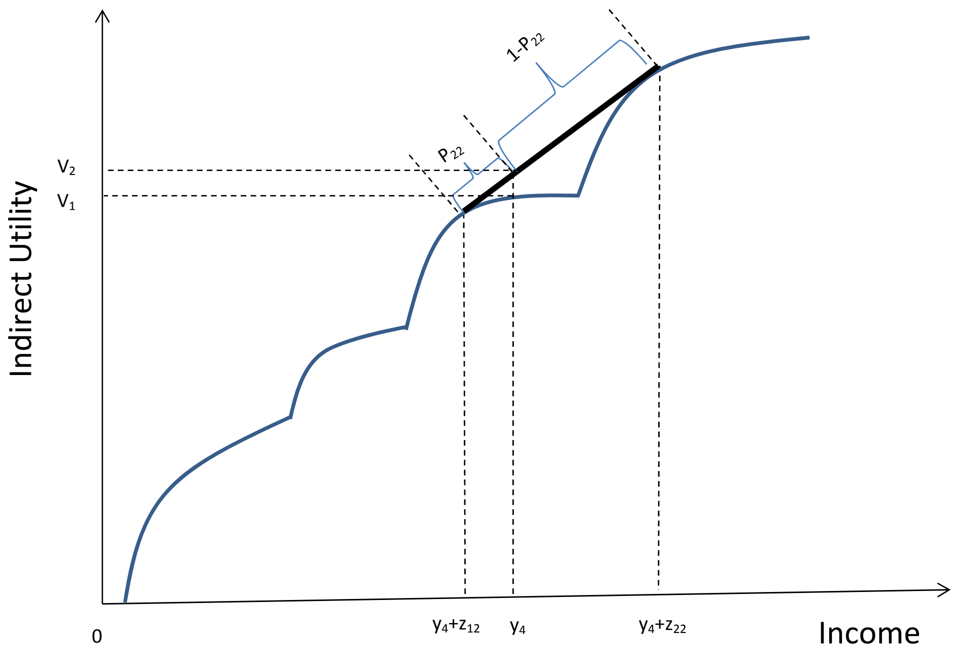

Figure 1.24: Lottery and imperfect income sorting, poorer household

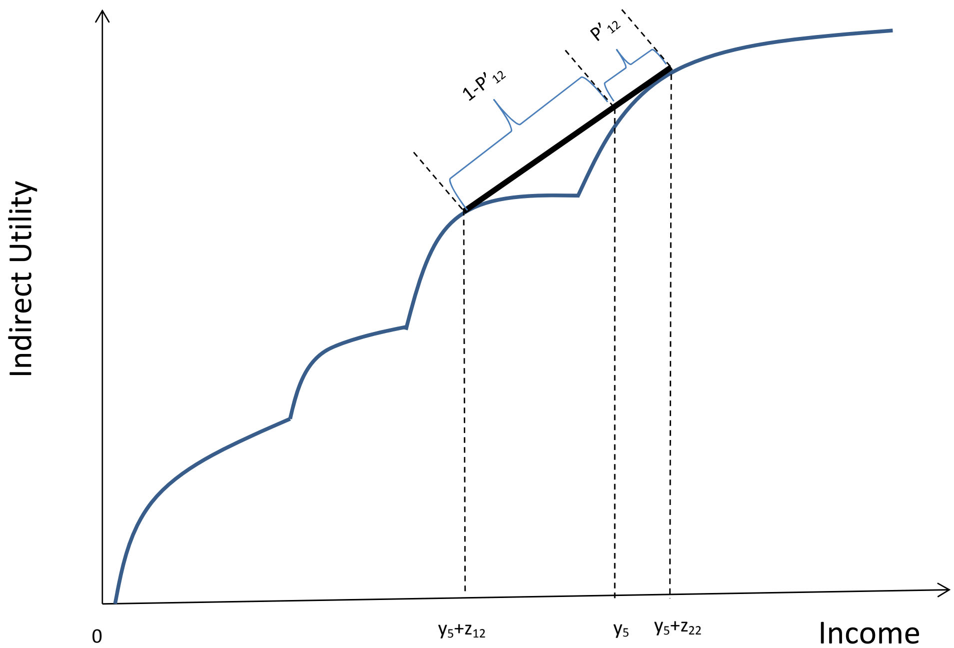

Figure 1.25: Lottery and imperfect income sorting, richer household

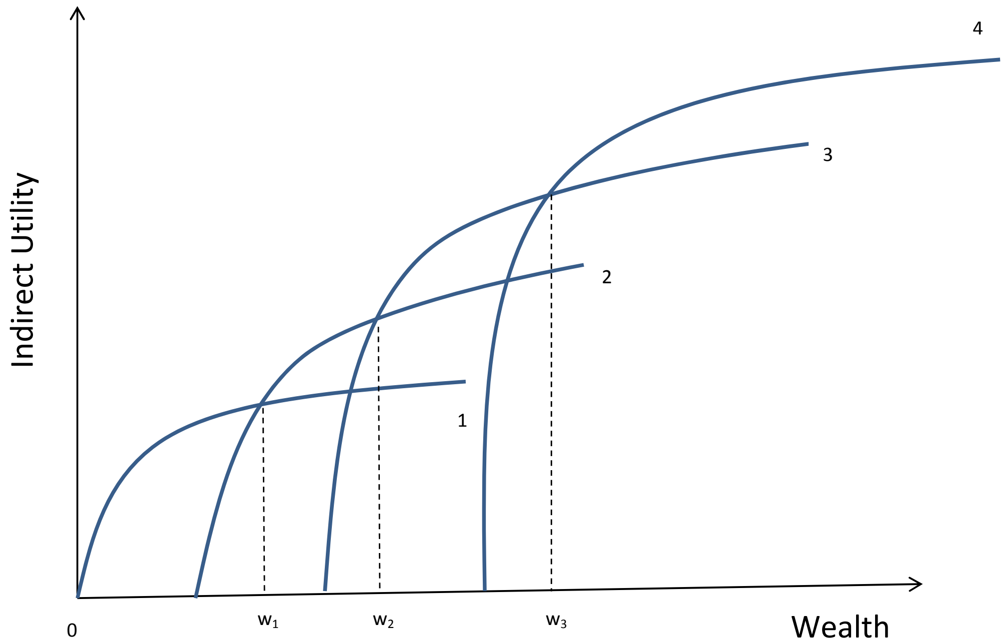

Figure 1.26: Perfect sorting w.r.t. wealth in Dunz (1985) and Nechyba (2003)

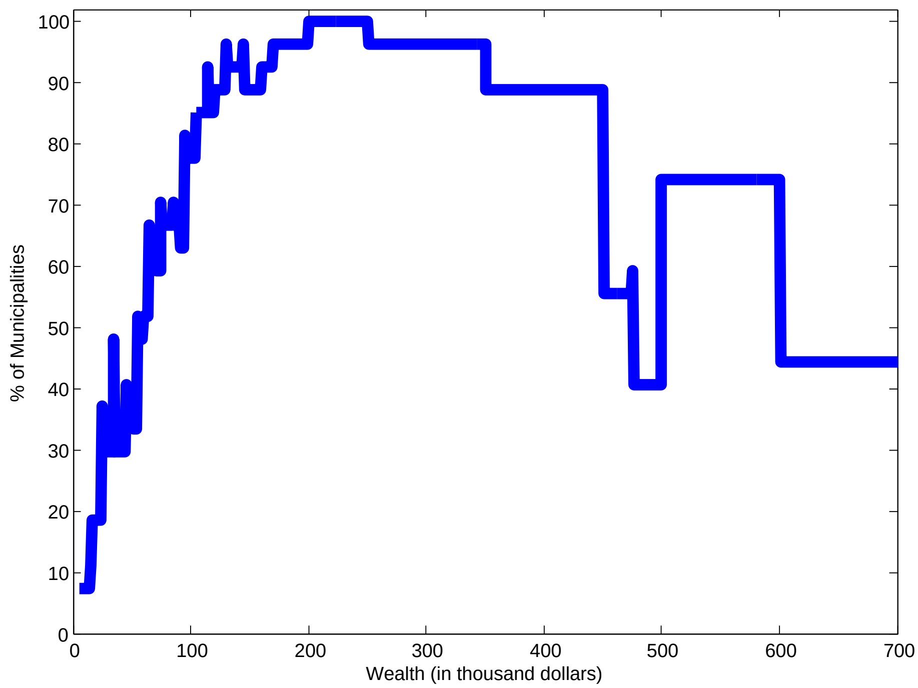

Figure 1.27: Imperfect wealth sorting in Rhode Island municipalities

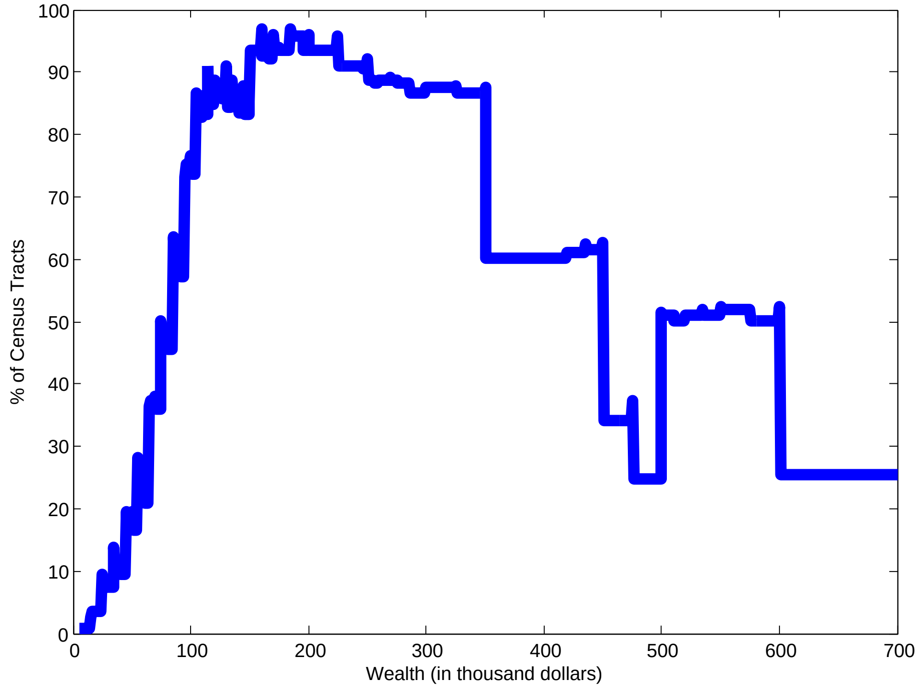

Figure 1.28: Imperfect wealth sorting in Rhode Island census tracts

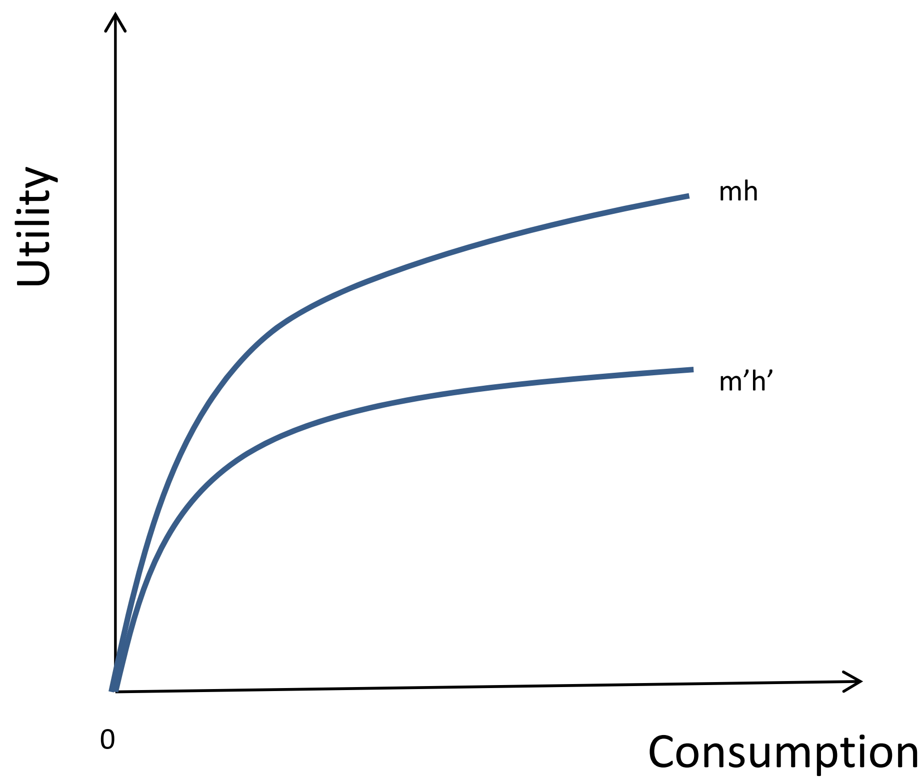

Figure 1.29: Illustration of assumptions 5 and 6

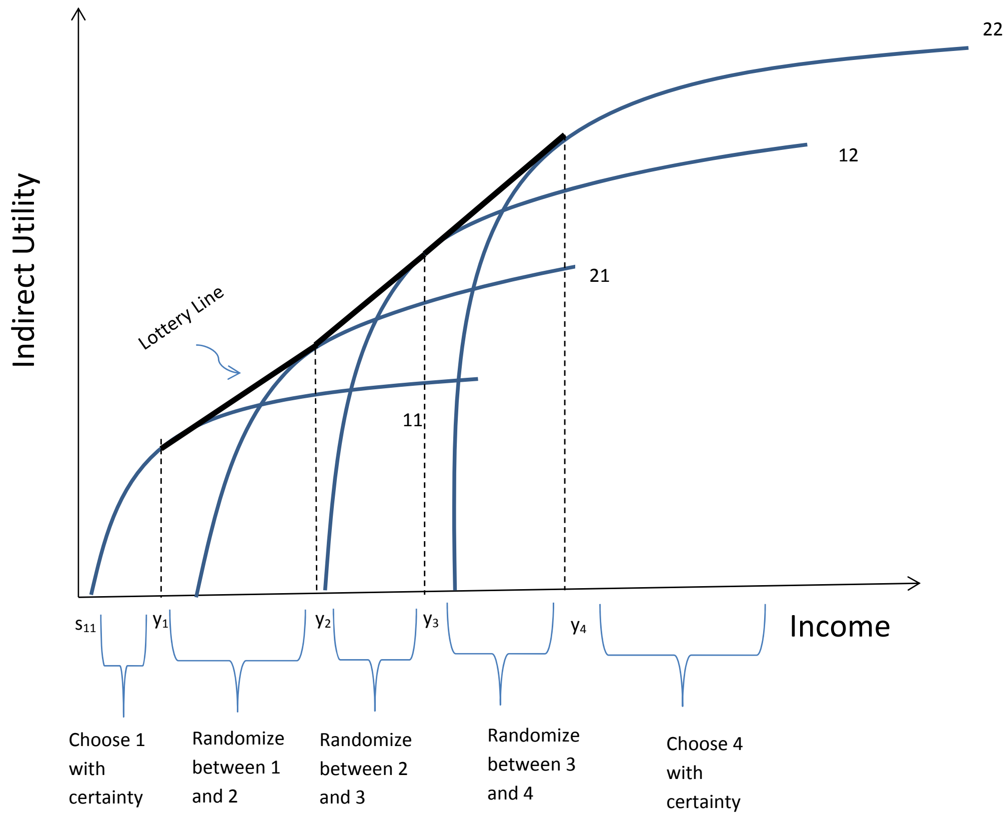

Figure 1.30: Household's lottery choice

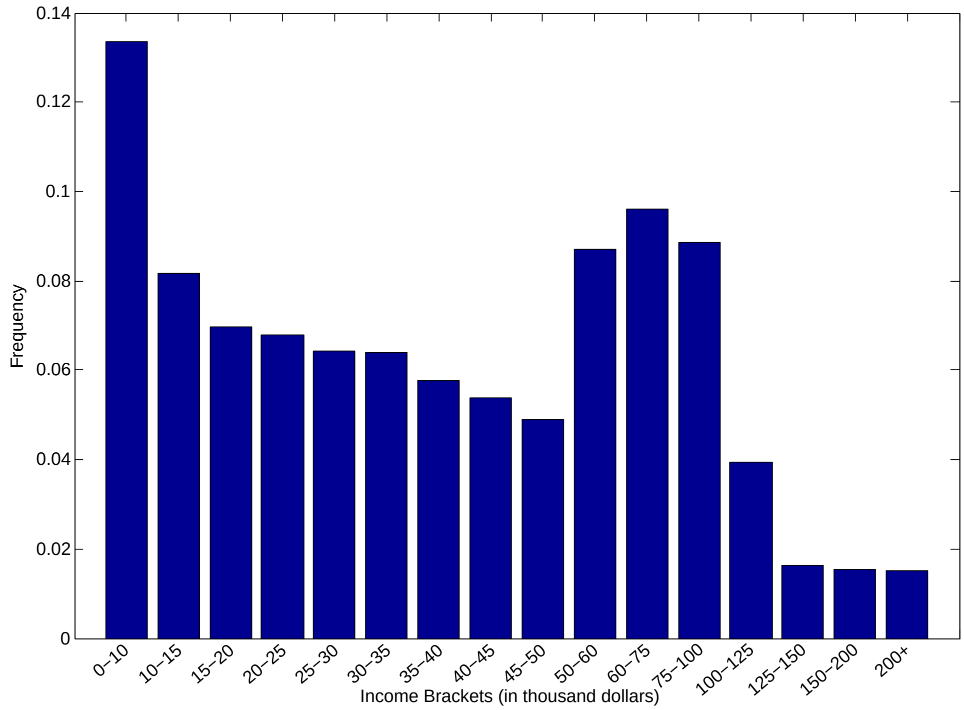

Figure 1.31: Histogram of income for Rhode Island

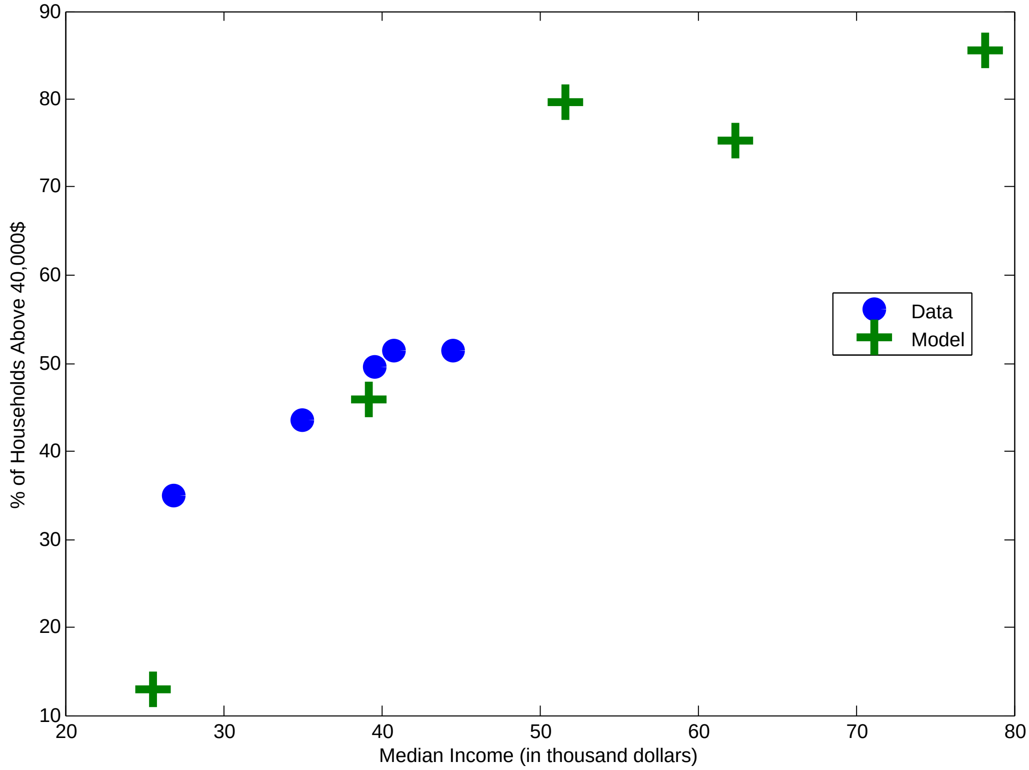

Figure 1.32: Imperfect income sorting across municipalities: Data vs. model

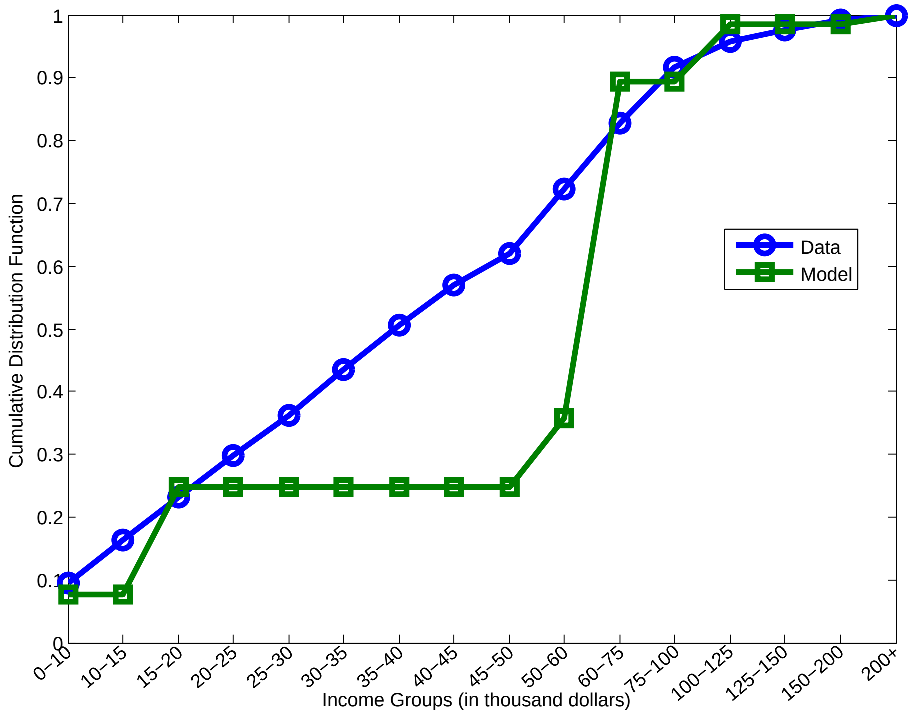

Figure 1.33: Income distribution in municipality I: Data vs. model

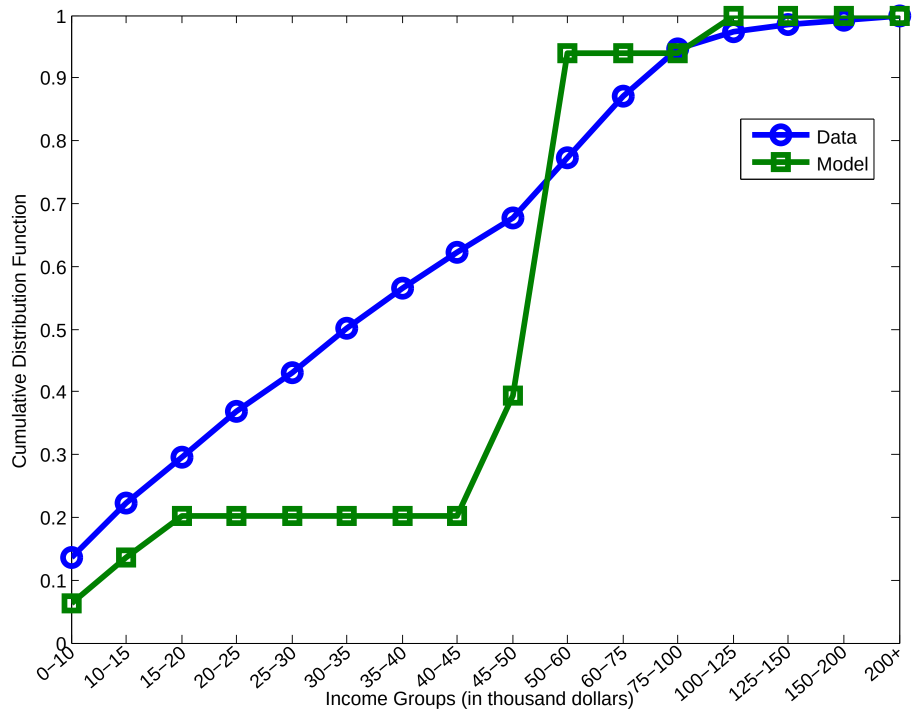

Figure 1.34: Income distribution in municipality II: Data vs. model

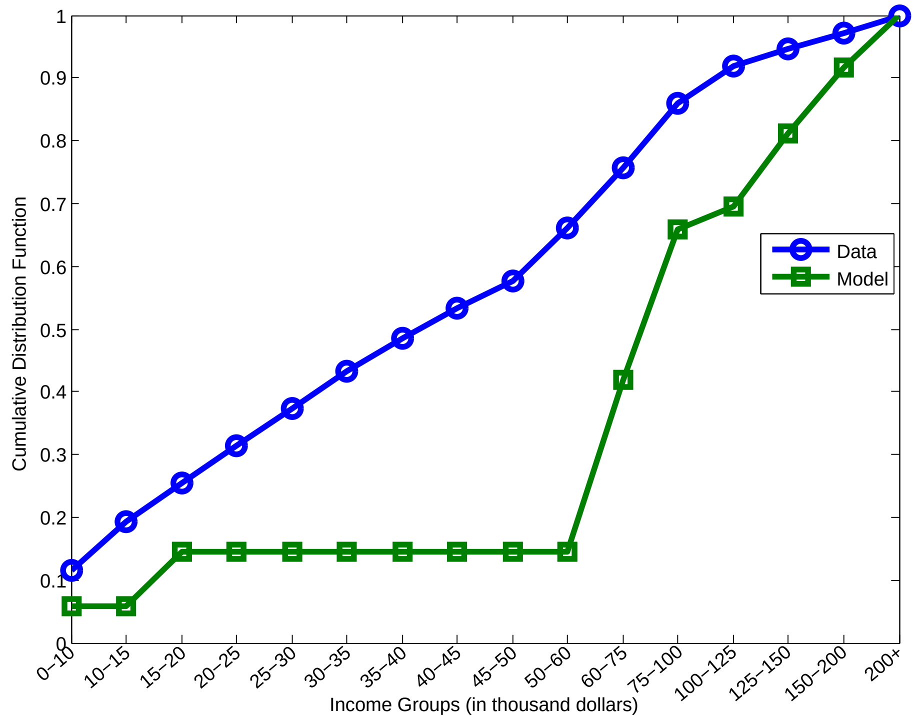

Figure 1.35: Income distribution in municipality III: Data vs. model

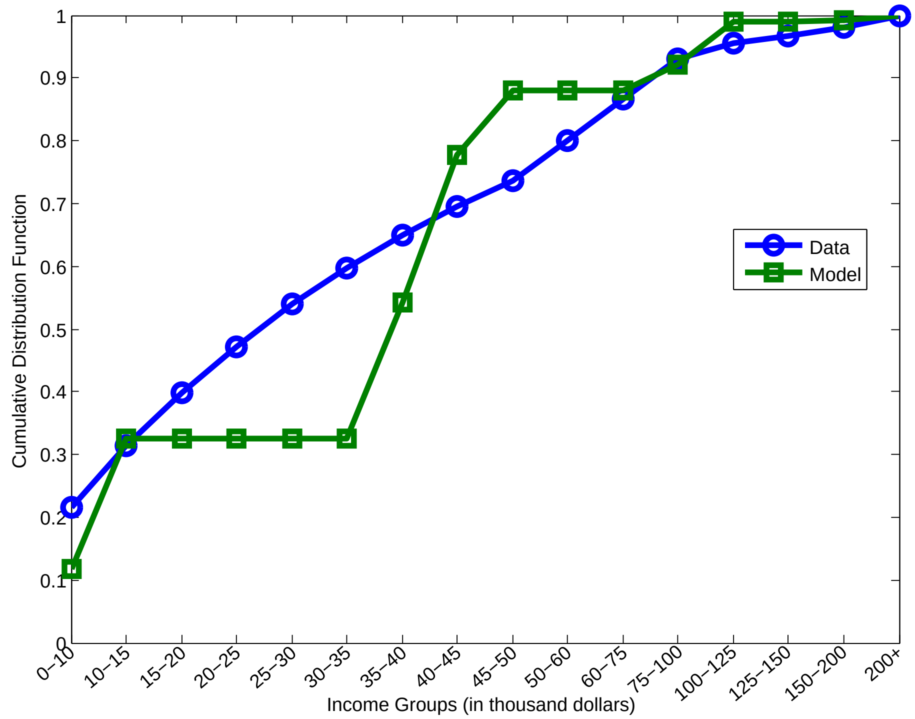

Figure 1.36: Income distribution in municipality IV: Data vs. model

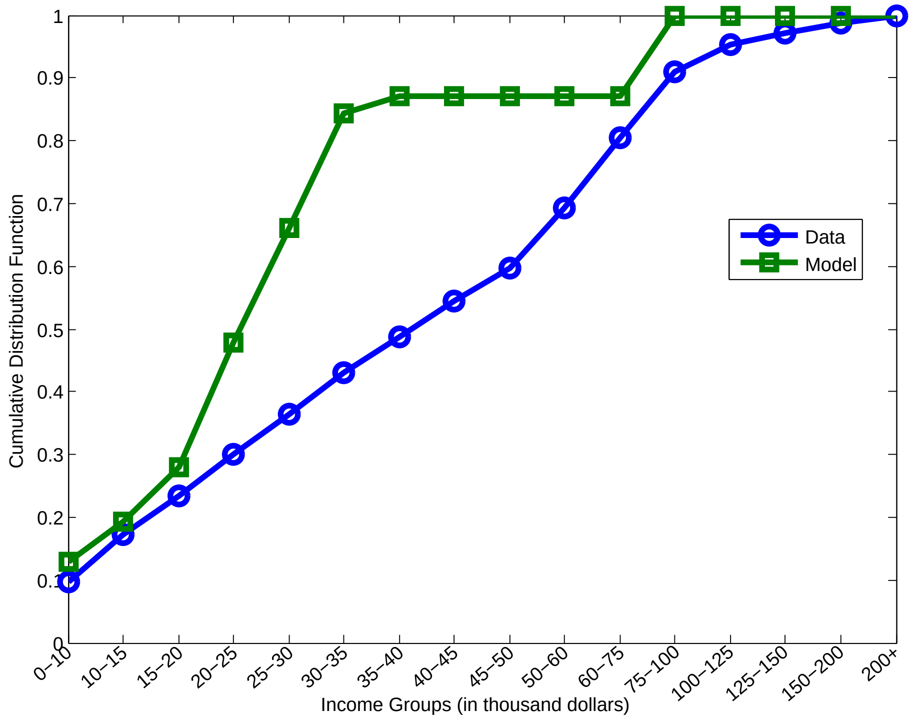

Figure 1.37: Income distribution in municipality V: Data vs. model

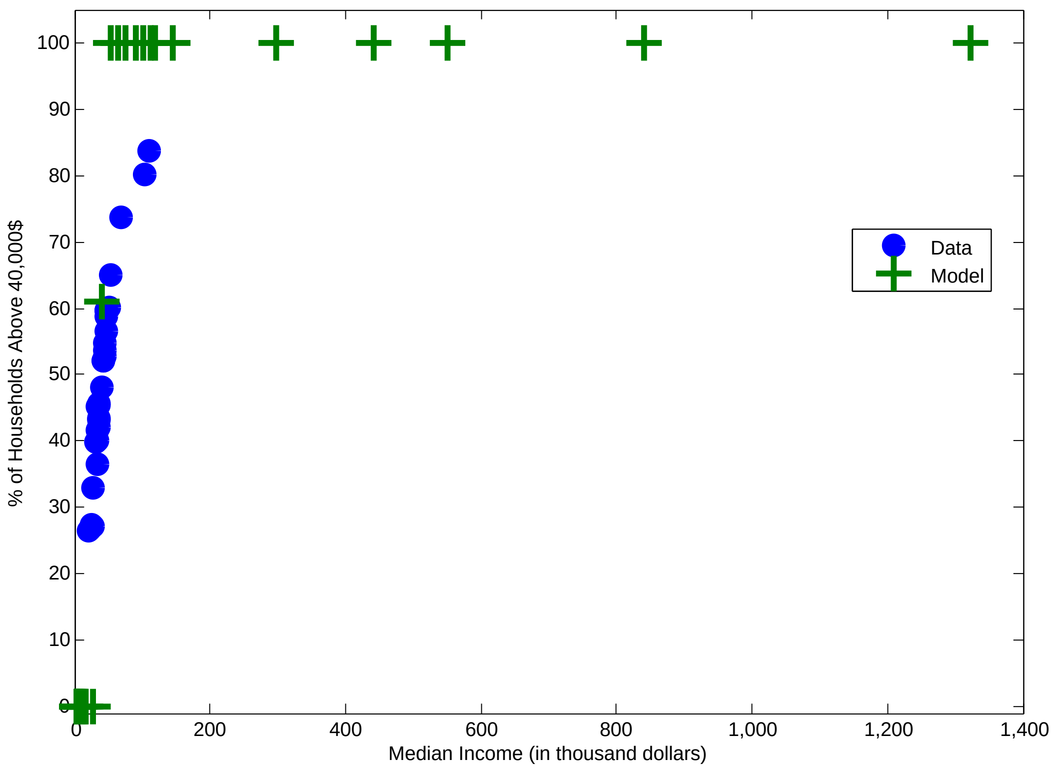

Figure 1.38: Imperfect income sorting across census tracts: Data vs. model

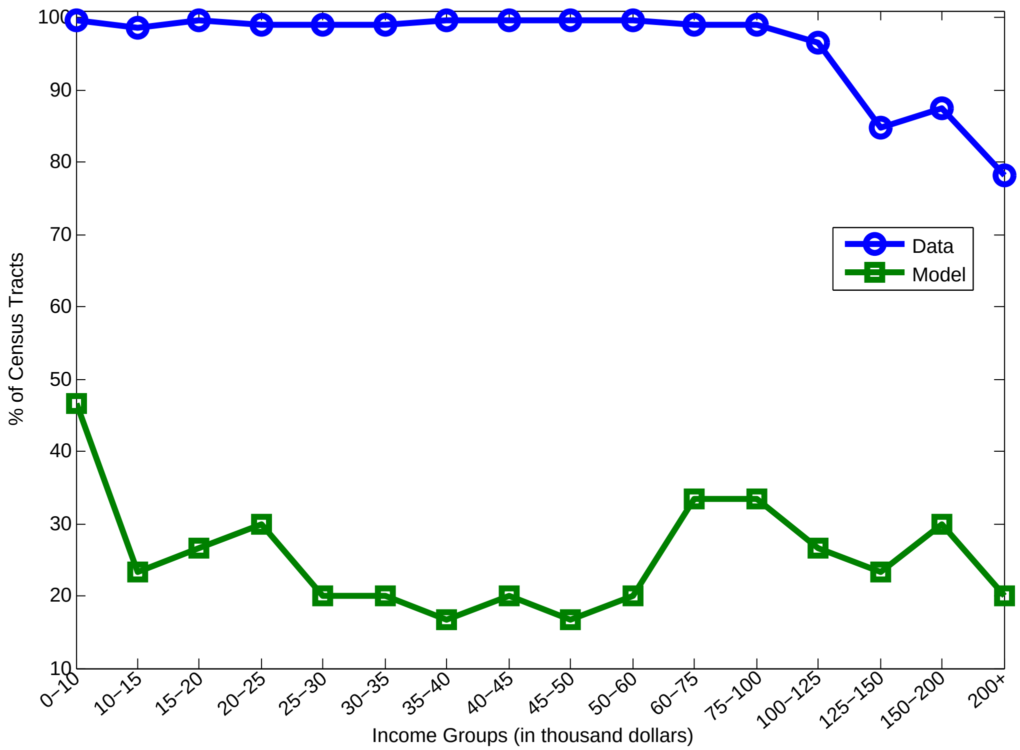

Figure 1.39: Another look at imperfect sorting across census tracts: Data vs. model

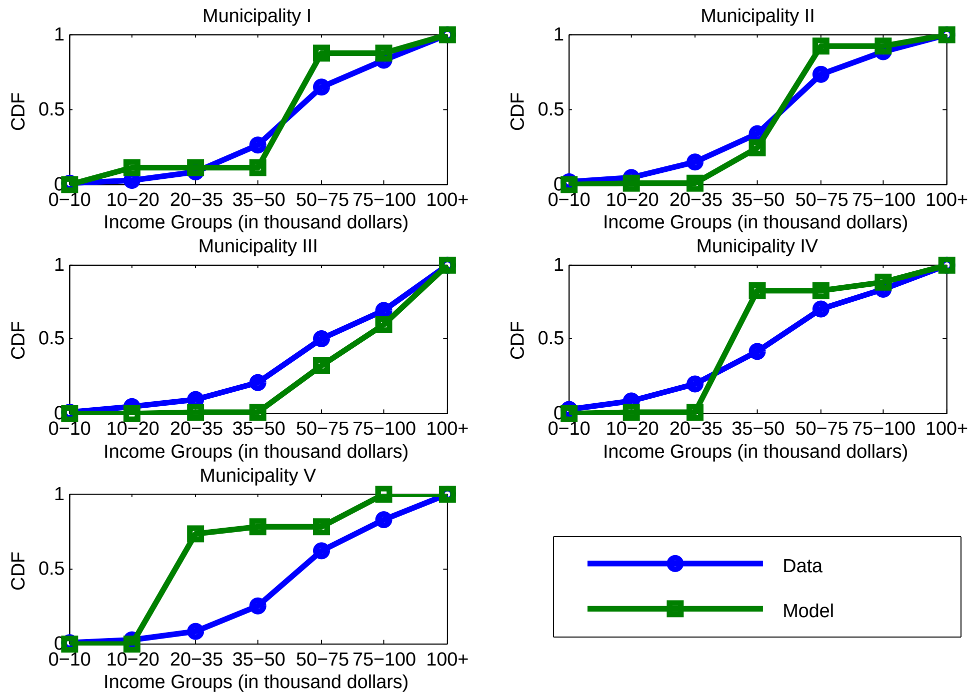

Figure 1.40: Imperfect income sorting conditional on rent share in income less than 14%: Data vs. model

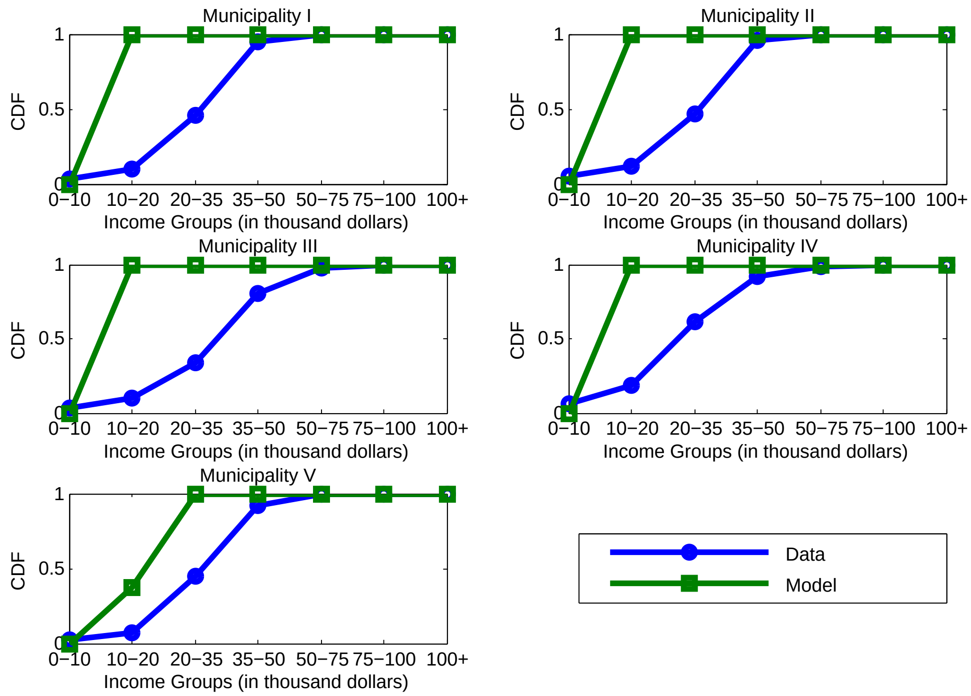

Figure 1.41: Imperfect Income sorting conditional on rent share in income between 14% and 19%: Data vs. model

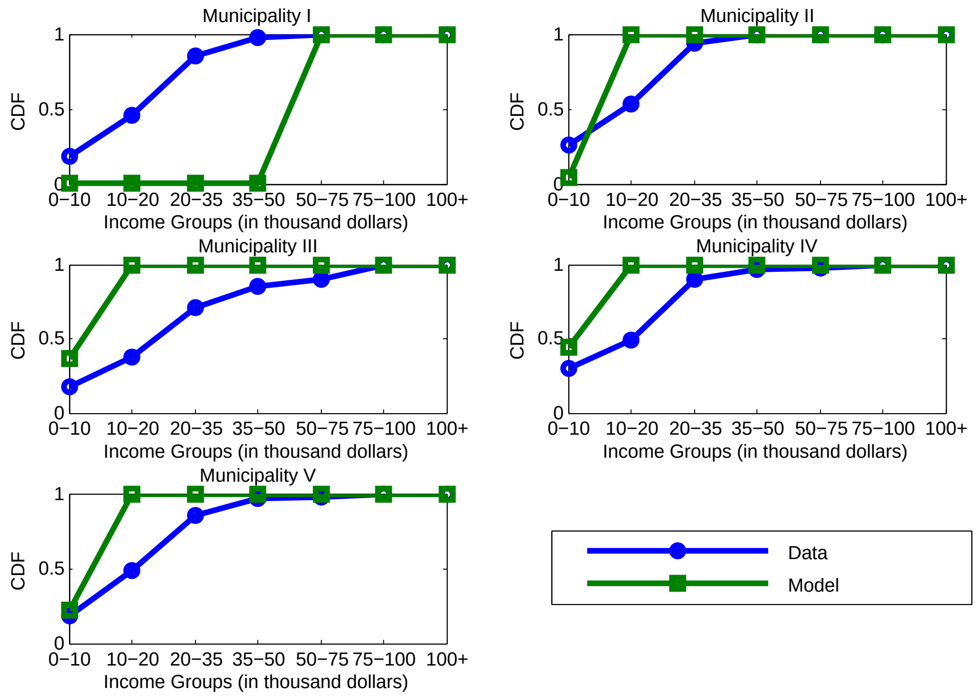

Figure 1.42: Imperfect Income sorting conditional on rent share in income between 19% and 24%: Data vs. model

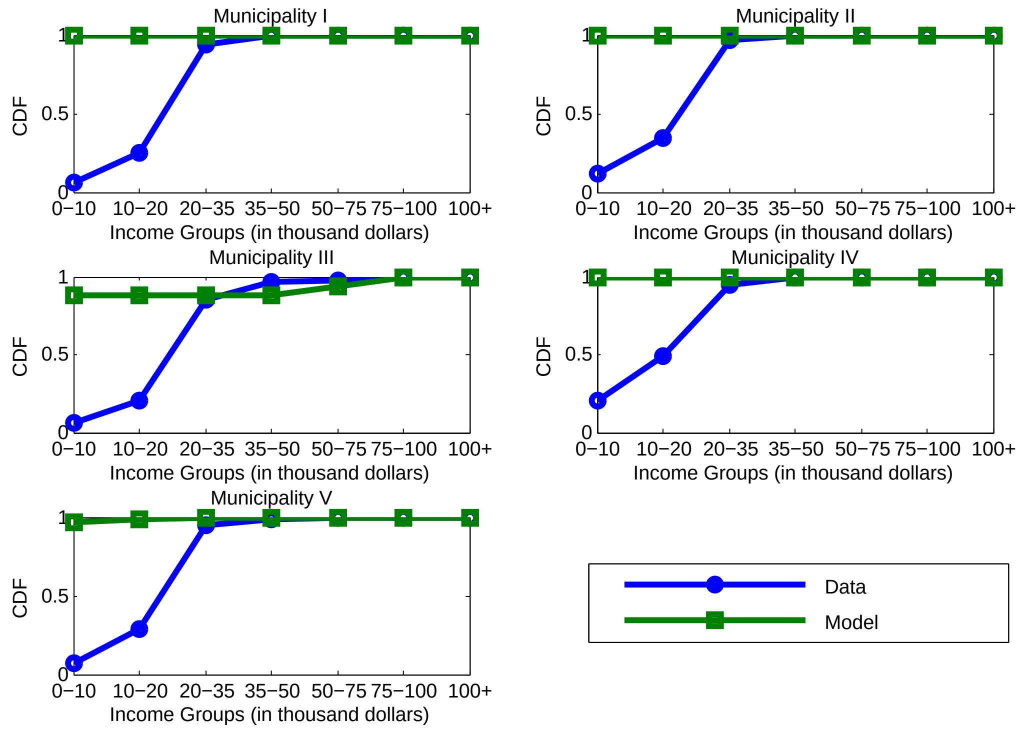

Figure 1.43: Imperfect income sorting conditional on rent share in income between 24% and 29%: Data vs. model

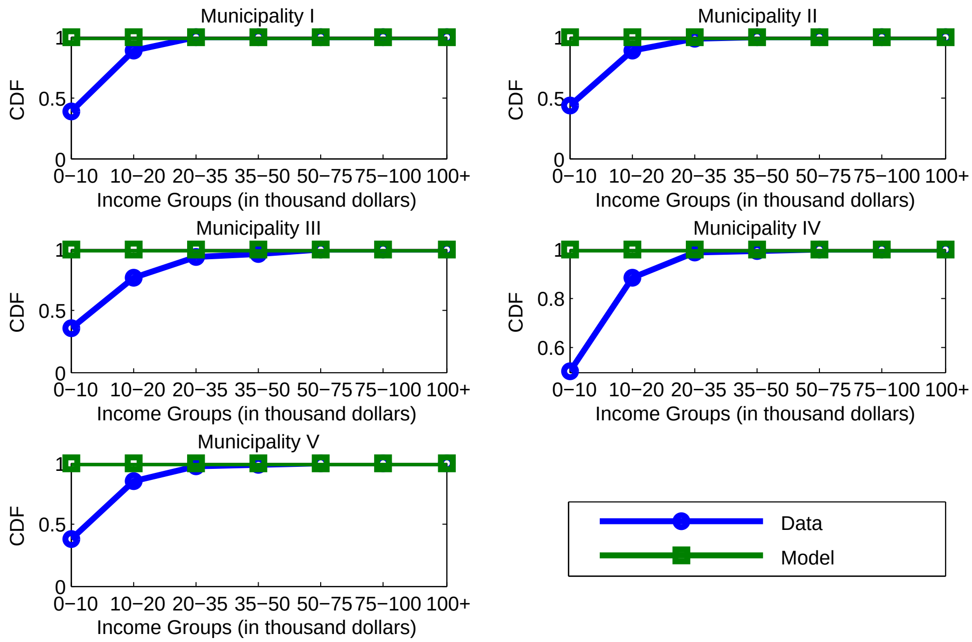

Figure 1.44: Imperfect income sorting conditional on rent share in income more than 29%: Data vs. model

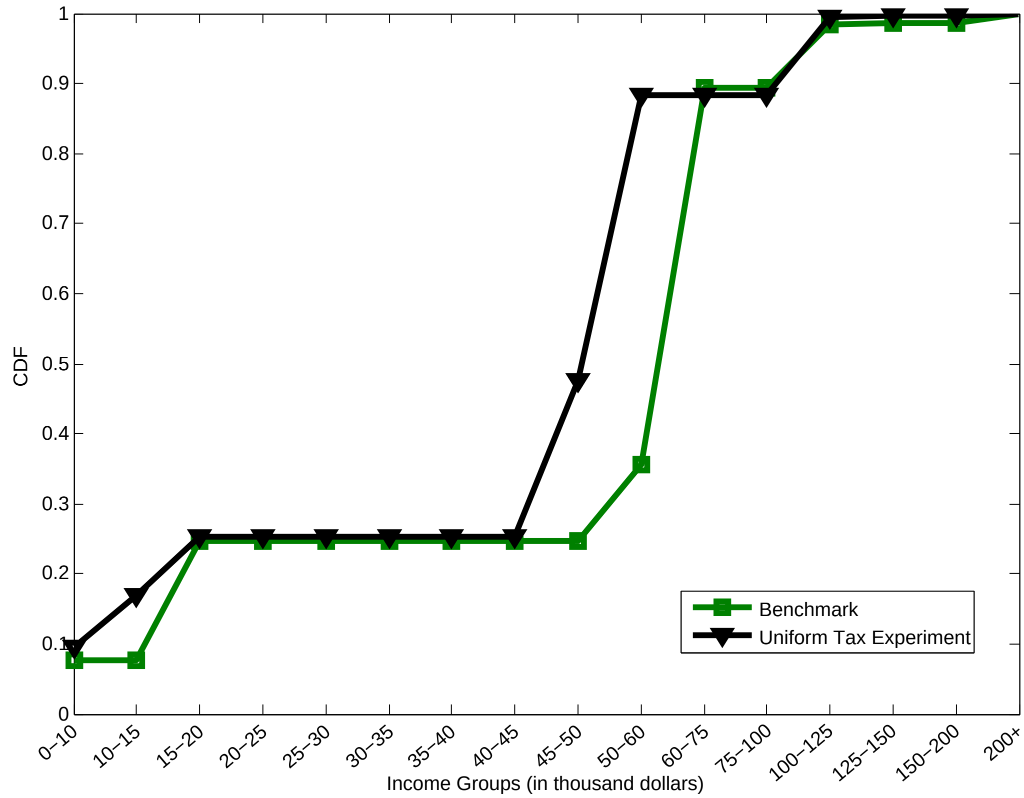

Figure 1.45: Income distribution in municipality I: Benchmark vs. uniform tax experiment

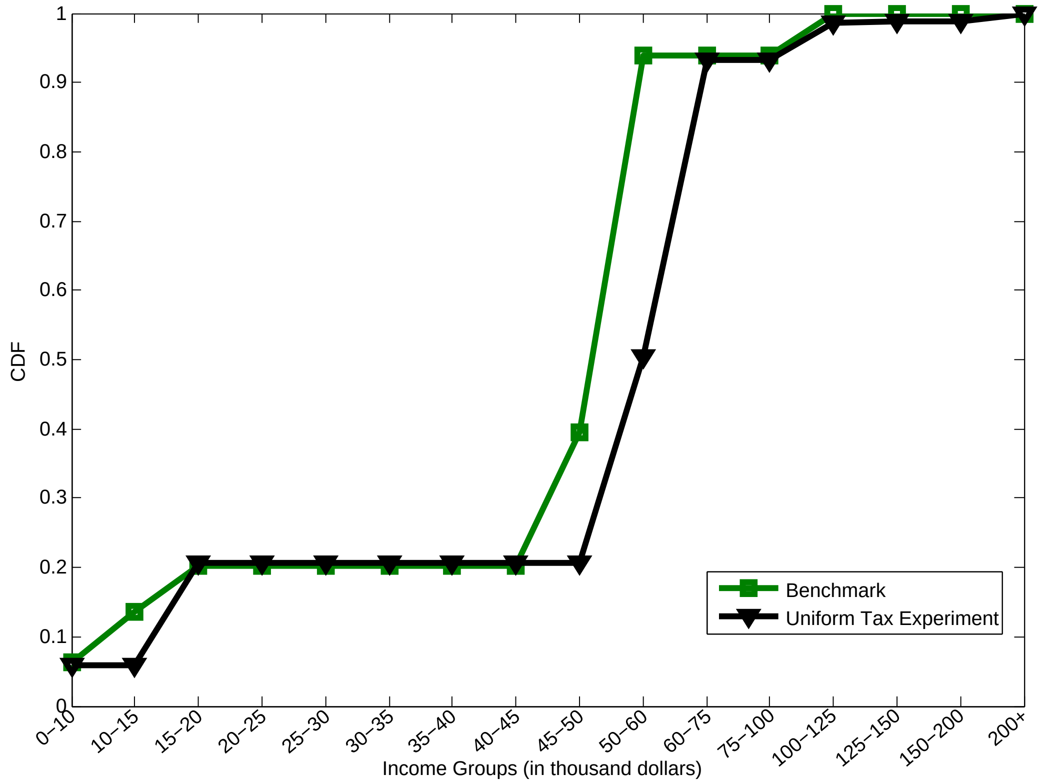

Figure 1.46: Income distribution in municipality II: Benchmark vs. uniform tax experiment

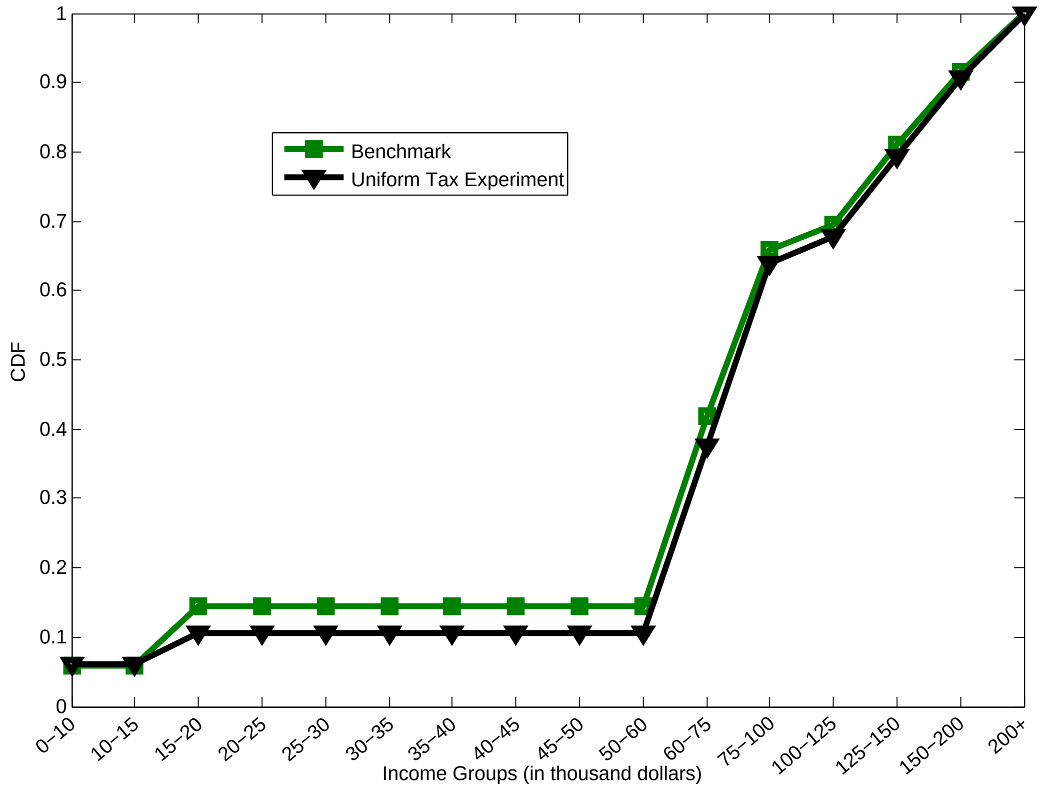

Figure 1.47: Income distribution in municipality III: Benchmark vs. uniform tax experiment

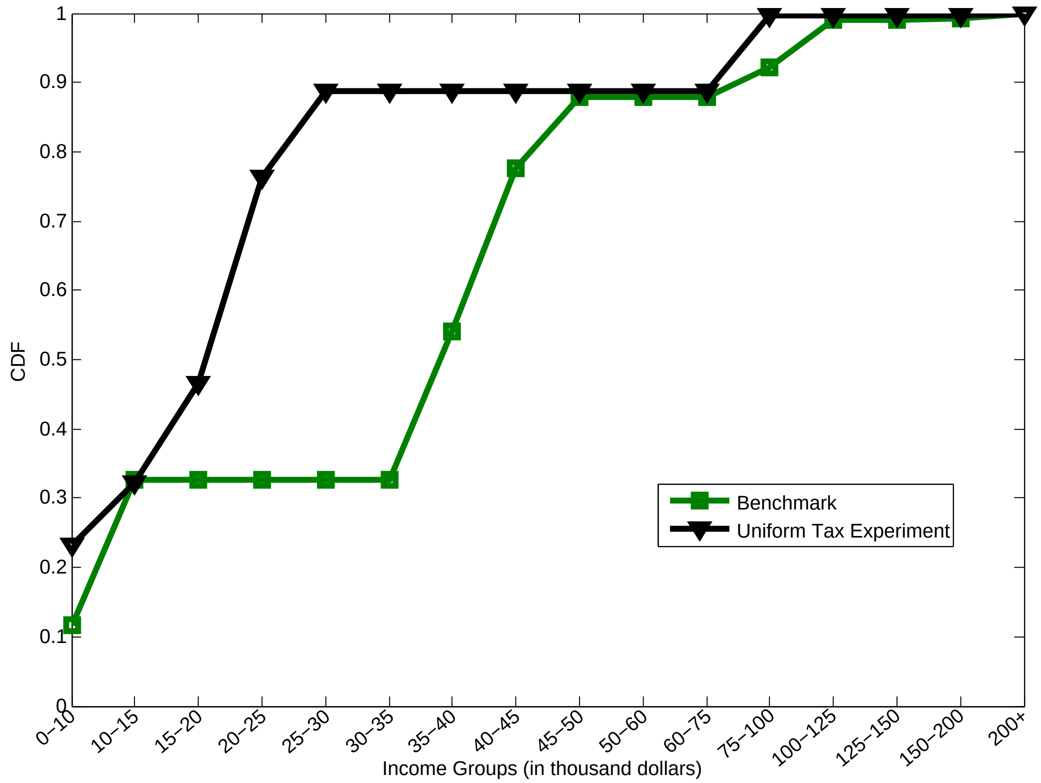

Figure 1.48: Income distribution in municipality IV: Benchmark vs. uniform tax experiment

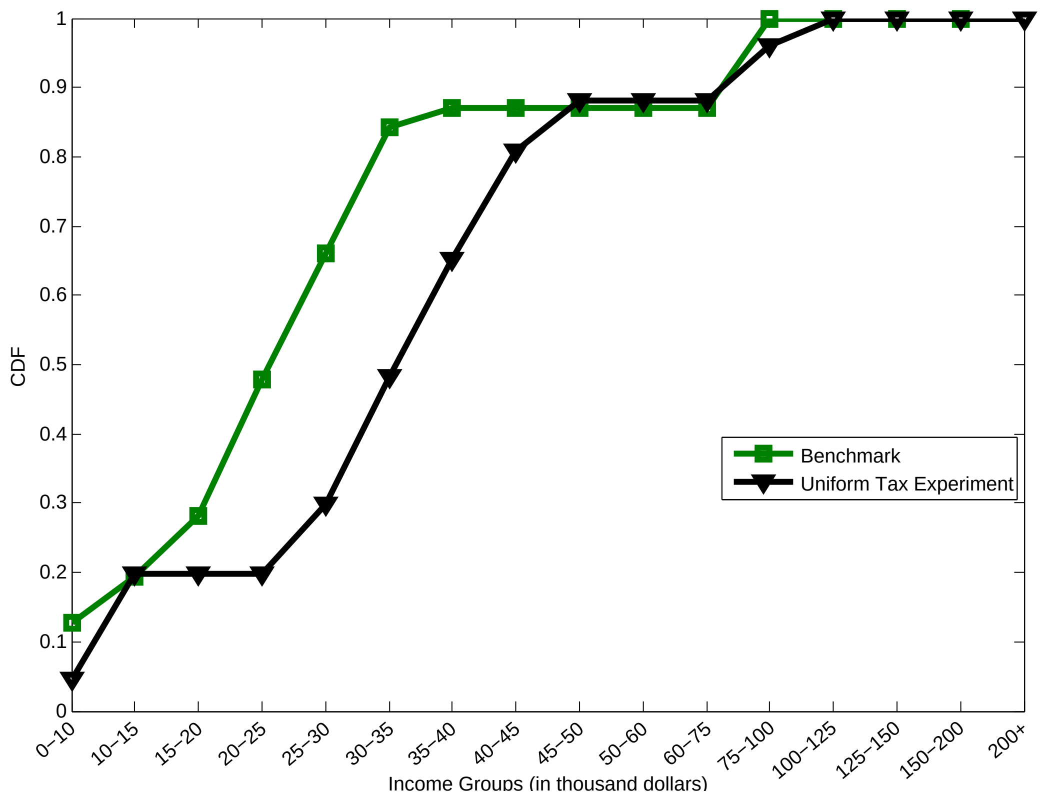

Figure 1.49: Income distribution in municipality V: Benchmark vs. uniform tax experiment

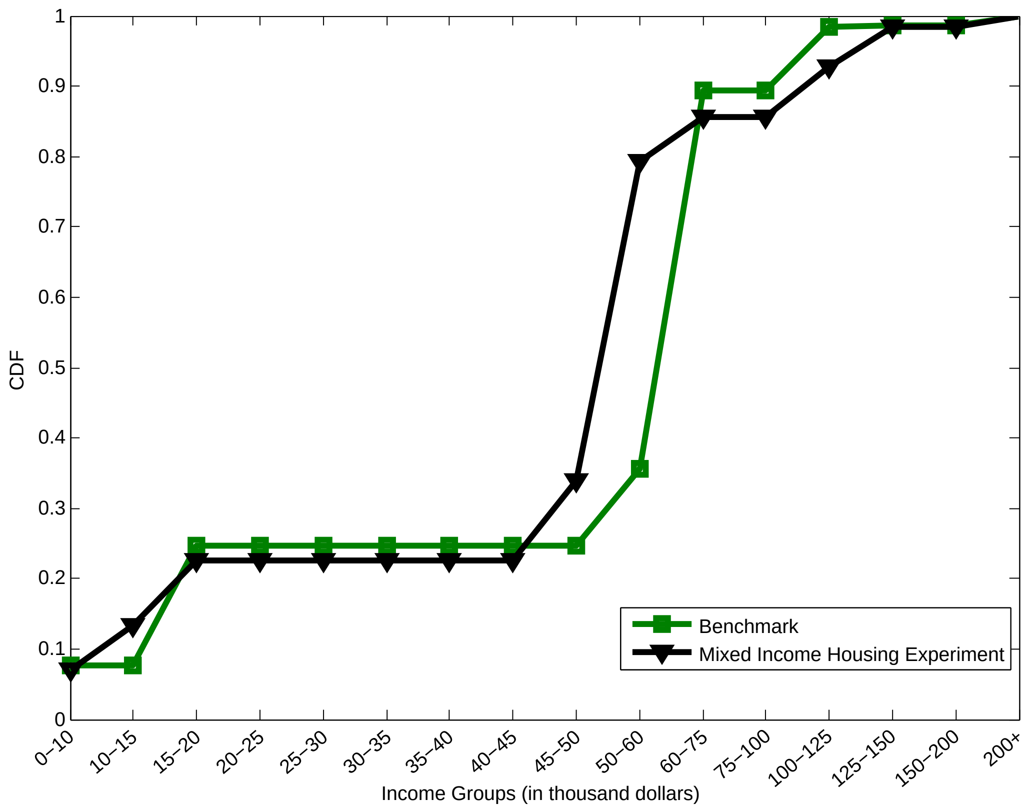

Figure 1.50: Income distribution in municipality I: Benchmark vs. mixed income housing experiment

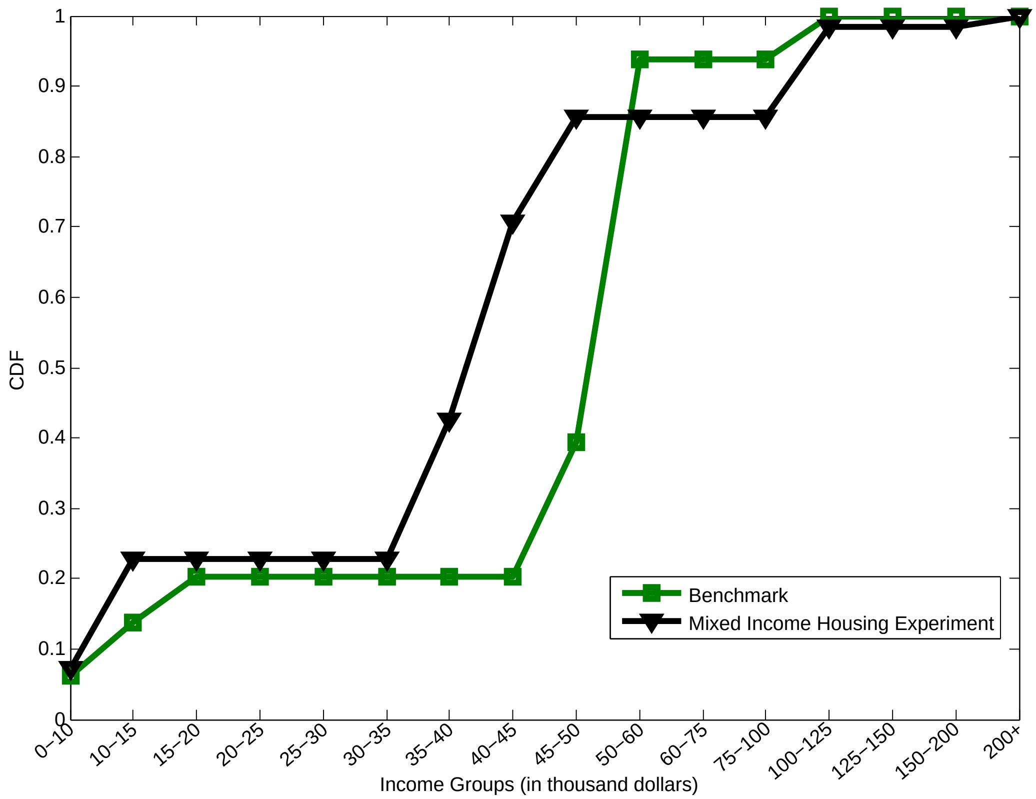

Figure 1.51: Income distribution in municipality II: Benchmark vs. mixed income housing experiment

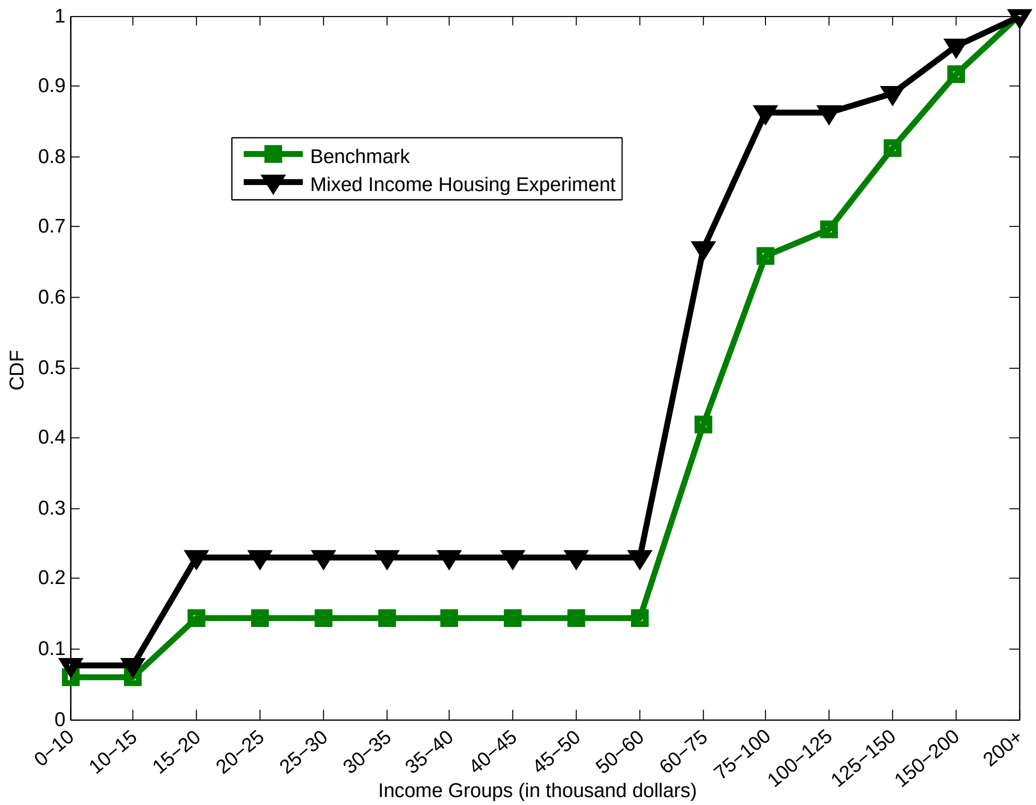

Figure 1.52: Income distribution in municipality III: Benchmark vs. mixed income housing experiment

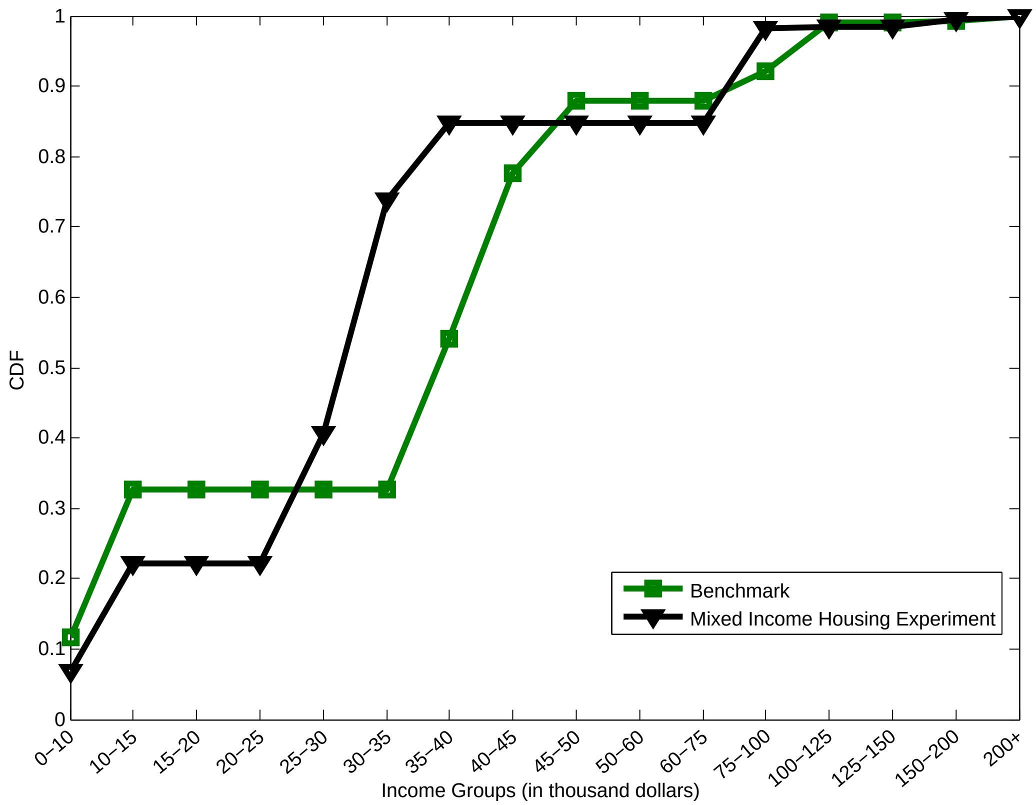

Figure 1.53: Income distribution in municipality IV: Benchmark vs. mixed income housing experiment

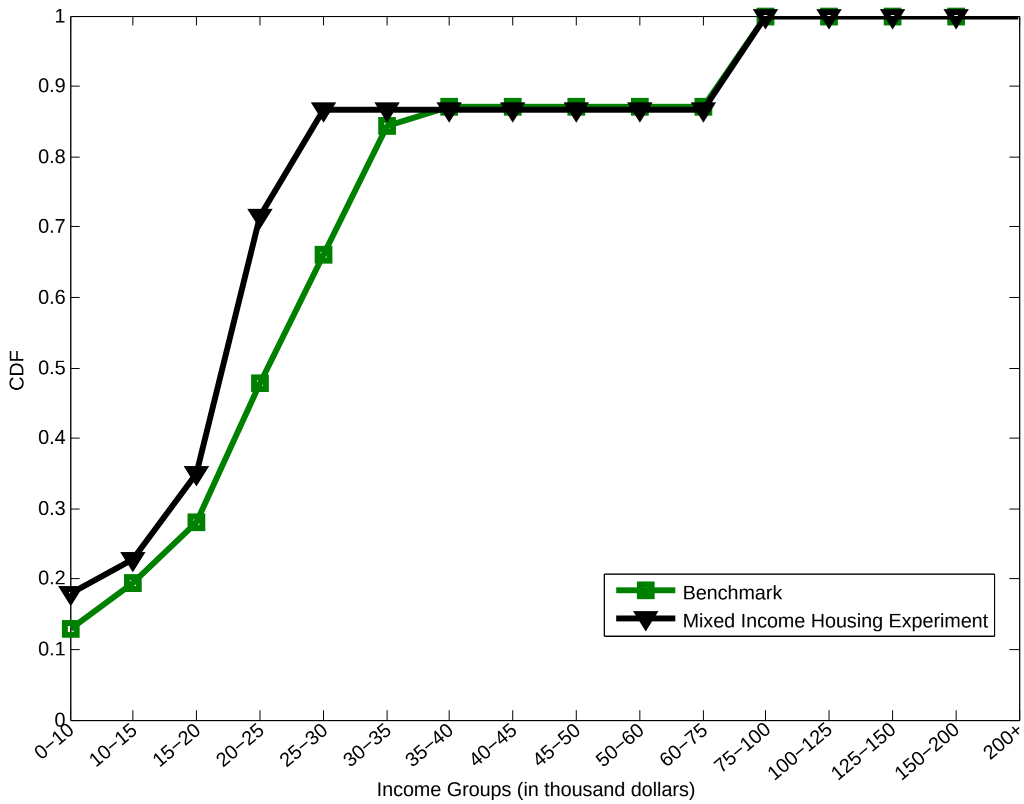

Figure 1.54: Income distribution in municipality V: Benchmark vs. mixed income housing experiment

CHAPTER 2
MEDIAN VOTER THEOREM FOR AN ECONOMY WITH PUBLIC AND MONOPOLISTICALLY COMPETITIVE PRIVATE SCHOOLS

2.1 Introduction

Throughout the world, education is supplied by both governments and private firms. Policy analysis for such economies proved difficult since majority voting equilibrium and decisive voter may not exist as argued in Stiglitz (1974) where private schools are modeled as perfectly competitive firms. However, empirical studies such as Bee and Dolton (1985), Christoffersen, Paldam, and Wurtz (2007), Kenny (1982), Kumar (1983) and Watt (1980) show that there are economies of scale in private schooling. Consistently with empirical evidence and differently from Stiglitz (1974), I model private schools as monopolistically competitive firms with decreasing average costs over enrollment in order to capture economies of scale. In my model economy, there does exist majority voting equilibrium and median income household is the decisive voter.

My model economy consists of households, private schools and a public school. Households are heterogeneous with respect to income and derive utility from numeraire consumption and the quality of education received by their child which is proxied by spending per pupil in the school in which they are enrolled. Households choose among a finite number of private schools and a public school. Each private school has a different quality and a different tuition. Per pupil spending in any private school simply equals the tuition which implies higher quality private schools are more expensive. The lowest quality private school has higher per pupil spending than the public school which is free. Public school spending is financed by income tax revenue collected from all households with the income tax rate determined by majority voting. Consumption of households that send their children to public school is equal to after tax income whereas it equals after tax income less tuition for those households that opt out of public school. Households therefore face a tradeoff between lower consumption and higher per pupil spending when choosing between public and private schools.

Because of economies of scale in private schools, as enrollment increases tuition charged per student decreases. I rationalize this by assuming the total cost of any private school has a fixed cost and a linear variable cost component. Therefore, the average cost is inversely related to enrollment. The real life counterpart of this fixed cost scenario is that setting up a private school requires land, teachers and equipment regardless of enrollment.

In the model, preferences of households over income tax rates turn out to be single peaked. The intuition for this result is as follows. When income tax rate is zero, households choose one of the private schools. When income tax rate is slightly above zero, enrollment increases in the public school and decreases in private schools. With lower enrollment private schools charge higher tuition because of economies of scale. Higher tuition also implies higher per pupil spending. Therefore, households that choose private schools consume less but enjoy higher per pupil spending. Under Cobb-Douglas utility combined with some assumptions on the model's parameters, the former effect is dominated by the latter effect. This implies the utility of households that choose private schools increases with increases in the income tax rate. Households continue to choose private schools until a sufficiently high income tax rate is reached. At or above that tax rate households choose public school. As the tax rate continues to rise, even though households choose public school, lower consumption begins to dominate the increases in per pupil public spending. Utility starts decreasing and becomes zero when the income tax rate becomes one. Once preferences are single peaked, existence of majority voting equilibrium follows from the theorem of Black (1948). Moreover, decisive voter also exists and is simply the median income household.

The paper is organized as follows. Section 2 summarizes relevant previous literature. Sections 3 explains the model. Section 4 provides the median voter theorem. Lastly section 5 concludes.

2.2 Previous literature

The theoretical analysis of an economy with both public and private schools is laid out by Stiglitz (1974). In this paper, private education is supplied by a continuum of perfectly competitive firms each of which has the same marginal cost. Any amount of education demanded by heterogeneous income households is supplied by these firms. As in my model, there is only one public school financed by income tax revenue. And the income tax rate is determined by majority voting among households. As in my model, income of a household is exogenously given and households receive utility from consumption and per pupil spending their child receives. In this economy, consumption of households that choose public school is simply equal to after tax income. Households that send their children to private school choose not only consumption but also the amount of private education. Consumption and private education spending for these households adds up to total income.

In Stiglitz (1974), household preferences over income tax rates are not single peaked. The intuition is as illustrated in Figure 2.1. At low income tax rates, households opt out of public education. Therefore, higher tax rates mean lower consumption and lower private education spending. This implies that utility decreases as tax rates increase. There exists a cutoff tax rate, t_1, at which a household is indifferent between public and private education. As the tax rate exceeds the cutoff, household finds it optimal to send its child to public school. When public school is chosen, utility increases as the tax rate increases since lower consumption is offset by higher per pupil public spending. There exists a tax rate, t_2, after which a decrease in utility from lower consumption dominates the increase in utility from higher per pupil public spending although household still chooses public school. Therefore there are two peaks in the households' preferences over income tax rates. Because of this, majority voting equilibrium may not exist and it is not straightforward to characterize the decisive voter.

Glomm and Ravikumar (1998) offer a solution to the nonexistence problem using the same model as Stiglitz (1974). Their analysis can be explained using Figure 2.2 which plots the indirect utility of four different income groups over income tax rates. The critical tax rate that makes median income household indifferent between public and private education is denoted by $\widehat{\tau}_m$. The interior peak when median income household chooses public school is denoted with τ_m. Their existence result relies on two propositions. The first one states that the critical tax rate is increasing in income. The second proposition numerically argues that, under specific functional forms for utility and income distribution, interior peak is decreasing in income. Using these two propositions, they prove that there exists a majority voting equilibrium and median income household is decisive voter.

Epple and Romano (1996) offer another solution to the nonexistence problem described in Stiglitz (1974). They analyze two different cases depending on the behavior of the slope of the utility function in the space of public spending and the tax rate. The two cases are that the slope is decreasing in income and increasing in income. Median voter theorem is provided for the first case only. As also noted in their paper, empirical studies provide evidence favoring the second case.

2.3 Model

I am considering a static economy which consists of a continuum of measure one households, R different quality private schools and a public school. Households are heterogeneous with respect to income and each has a school age kid that attends either a private school or public school. Public schools are financed out of mandatory income taxes paid by households. Public schools don't charge a tuition whereas private school's expenditure is financed by tuition payments. Each private school has a different tuition reflecting quality differences. Moreover, as will become clearer in the subsequent sections, I assume a monopolistically competitive market structure for the private schools.

2.3.1 Preferences

Households have identical preferences defined over (c, q) pairs in $\Re^2_+$ where (c, q) represents consumption of the numeraire good and per pupil spending in the school that the household's kid attends. Preferences are represented by a Cobb-Douglas utility form as follows:

$$u(c, q) = c^{\alpha} q^{\beta}$$

where $\alpha > 0$ and $\beta > 0$.

2.3.2 Endowments

Households are heterogeneous with respect to exogenous receipts of income y measured in terms of numeraire consumption and income is distributed according to some cumulative distribution function $F(\cdot)$ with support $\Re_+$. The mean income in the economy is denoted with Y which is also equal to total income in the economy since measure of households is one. The standard deviation of $F(\cdot)$ is σ.

2.3.3 Schools

2.3.3.1 Public schools

There is only one public school at which the per pupil spending is equal to q_u. Public school expenditures is financed by the income tax revenue. Public school budget is:

$$N_u q_u = \tau Y \tag{2.1}$$

where τ is the income tax rate and N_u is the enrollment in public school. Since total mass of students is one, N_u is also the fraction of total households sending kid to public school. Left hand side of (2.1) is total spending on students and right hand side is total income tax revenue.

2.3.3.2 Private schools

There are R private schools in the economy. Enrollment in the private school r is denoted with N_r. Tuition in private school r is p_r. Private schools behave as monopolistically competitive firms. Assuming a total cost function consisting of a fixed cost and a variable cost proportional to output in the very basic model of Dixit and Stiglitz (1977) with both consumers and firms implies the following reduced form relation between tuition and enrollment:

$$p_r N_r = b_r \tag{2.2}$$

where b_r is a parameter specific to private school r capturing both cost differences across private schools and also consumers' preference parameters. The implication of this particular relation between enrollment and tuition is that as enrollment goes down tuition goes up. The intuition behind this is that since fixed cost does not adjust as enrollment goes down, the private school has to increase tuition charged per student. For instance, the private school has to pay the same amount of rent for land or hire a minimum amount of teachers from each field regardless of the enrollment.

The budget constraint of the private school r implies that tuition per student, p_r, is equal to per pupil spending q_r:

$$q_r = p_r \tag{2.3}$$

Moreover, I assume per pupil spending in private school r is some multiple of the per pupil spending in the public school. Mathematically the following relation holds:

$$q_r = \gamma_r q_u \tag{2.4}$$

where γ_r is a parameter satisfying:

$$1 < \gamma_1 < \gamma_2 < ... < \gamma_R$$

Therefore knowledge of q_u is enough to determine N_r.

2.3.4 Household's decision problem

Household with a particular income y compares indirect utilities from choosing public school or one of the private schools. The indirect utility from public school is denoted by V_u and indirect utility from private school r is denoted by V_r.

If household chooses public school, the following problem is solved given y, τ and N_u:

$$V_u(\tau, N_u, y) = \max_{c_u} u(c_u, q_u) \tag{2.5}$$

$$s.t. \qquad c_u = (1-\tau)y$$

$$q_u = \frac{\tau Y}{N_u}$$

On the other hand, if household chooses private school r, the following problem pins down V_r^y given y, τ and N_u:

$$V_r(\tau, N_u, y) = \max_{c_r} u(c_r, q_r) \tag{2.6}$$

$$s.t. \qquad c_r + p_r = (1-\tau)y$$

$$q_r = p_r$$

$$q_r = \gamma_r \frac{\tau Y}{N_u}$$

The final value received by household with income y is equal to:

$$V(\tau, N_u, y) = \max\{V_u(\tau, N_u, y), V_1(\tau, N_u, y), V_2(\tau, N_u, y), ..., V_R(\tau, N_u, y)\} \tag{2.7}$$

2.3.5 Majority voting equilibrium

A majority voting equilibrium is a collection of consumptions for each household $\{c_u^{y*}, (c_r^{y*})_{r=1}^R\}$, tuition p_r^* for each private school r, per pupil spending in public and private schools $\{q_u^*, (q_r^*)_{r=1}^R\}$, enrollment in public and private schools $\{N_u^*, (N_r^*)_{r=1}^R\}$, income tax rate τ^* and indicator function

$I_r(V(\tau^*, N_u^*, y) = V_r(\tau^*, N_u^*, y))$ for each private school r and each household such that:

1. $\{c_u^{y*}, (c_r^{y*})_{r=1}^R\}$ solves the household problems (2.5) and (2.6).

2. τ^* and N_u^* satisfies the following:

   (a) Given τ^*, N_u^* solves:

   $$N_u^* = \int_0^1 [1 - \sum_{r=1}^R I_r(V(\tau^*, N_u^*, y) = V_r(\tau^*, N_u^*, y))]dF(y)$$

   (b) There does not exist another pair $\{\tau^{'}, N_u^{'}\}$ such that:

   - Given $\tau^{'}$, $N_u^{'}$ solves:

   $$N_u^{'} = \int_0^1 [1 - \sum_{r=1}^R I_r(V(\tau^{'}, N_u^{'}, y) = V_r(\tau^{'}, N_u^{'}, y))]dF(y)$$

   - At least half of the households prefers $\tau^{'}$ over τ^*.

3. Private schools are monopolistically competitive:

$$p_r N_r = b_r$$

4. Budget of the public school and private school r is balanced:

$$N_u q_u = \tau Y$$

$$q_r = p_r \;\; \forall r$$

5. Per pupil spending in private school r is a multiple of per pupil public spending:

$$q_r = \gamma_r q_u$$

6. Total enrollment in public and private schools is equal to total household population:

$$N_u^* + \sum_{r=1}^{R} N_r^* = 1$$

2.4 Existence of majority voting equilibrium

In this section, I will prove that household's preference over income tax rates is single peaked. This will allow us to prove existence of majority voting equilibrium following Black (1948). Moreover, I will also prove that median income household is the decisive voter.

Lemma 2. *The following relations hold:*

$$N_u = \frac{\tau Y}{\tau Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}$$

$$N_r = \frac{b_r}{\gamma_r \tau Y + \gamma_r \sum_{r=1}^{R} \frac{b_r}{\gamma_r}} \quad \forall r$$

$$q_u = \tau Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}$$

$$p_r = q_r = \gamma_r \tau Y + \gamma_r \sum_{r=1}^{R} \frac{b_r}{\gamma_r} \quad \forall r$$

Proof. By (2.3) and (2.4):

$$p_r = \gamma_r q_u$$

Substituting this into (2.2) gives:

$$\gamma_r N_r q_u = b_r$$

Substituting for q_u gives:

$$N_r = \frac{b_r N_u}{\gamma_r \tau Y} \tag{2.8}$$

Substituting N_r into the last condition in the equilibrium definition gives:

$$N_u = \frac{\tau Y}{\tau Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}$$

Substituting N_u into (2.8) gives:

$$N_r = \frac{b_r}{\gamma_r \tau Y + \gamma_r \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}$$

Moreover;

$$q_u = \frac{\tau Y}{N_u} = \tau Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}$$

$$p_r = q_r = \gamma_r q_u = \gamma_r \tau Y + \gamma_r \sum_{r=1}^{R} \frac{b_r}{\gamma_r}$$

□

Consider the income y household's problem (2.5) choosing public school. Since $c_u = (1-\tau)y$ and q_u is as given in Lemma 2 then indirect utility from public school is:

$$V_u(\tau, y) = u(c_u, q_u) = ((1-\tau)y)^{\alpha} \left(\tau Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r} \right)^{\beta}$$

Similarly for an income y household choosing private school r, consumption is $c_r = (1-\tau)y - \tau Y \gamma_r - \gamma_r \sum_{r=1}^{R} \frac{b_r}{\gamma_r}$ and q_r is given by Lemma 2. Hence the indirect utility from private school r is:

$$V_r(\tau, y) = u(c_r, q_r) = \left((1-\tau)y - \tau Y \gamma_r - \gamma_r \sum_{r=1}^{R} \frac{b_r}{\gamma_r} \right)^{\alpha} \left(\gamma_r \tau Y + \gamma_r \sum_{r=1}^{R} \frac{b_r}{\gamma_r} \right)^{\beta}$$

Proposition 2. *$V_u(\tau, y)$ and $V_r(\tau, y)$ are strictly concave in τ.*

Proof.

$$\frac{\partial V_u}{\partial \tau} = -u_c(c_u, q_u)y + u_q(c_u, q_u)Y$$

$$\frac{\partial^2 V_u}{\partial \tau^2} = -y[\underbrace{-u_{cc}(c_u, q_u)y}_{>0} + \underbrace{u_{cq}(c_u, q_u)\tau}_{>0}] + Y[\underbrace{-u_{cq}(c_u, q_u)y}_{<0} + \underbrace{u_{qq}(c_u, q_u)\tau}_{<0}] < 0$$

$$\frac{\partial V_r}{\partial \tau} = -u_c(c_r, q_r)(y + Y\gamma_r) + u_q(c_r, q_r)Y\gamma_r$$

$$\frac{\partial^2 V_r}{\partial \tau^2} = -(y + Y\gamma_r)[\underbrace{-u_{cc}(c_r, q_r)(y + Y\gamma_r)}_{>0} + \underbrace{u_{cq}(c_r, q_r)Y\gamma_r}_{>0}] +$$

$$Y\gamma_r[\underbrace{-u_{cq}(c_r, q_r)(y + Y\gamma_r)}_{<0} + \underbrace{u_{qq}(c_r, q_r)Y\gamma_r}_{<0}] < 0$$

□

Definition 4. *The most preferred tax rate for a household choosing public school is defined as:*

$$\tau_u = \arg\max_{\tau} V_u(\tau, y)$$

and the utility level at this peak is:

$$V_u^p(y) = \max_{\tau} V_u(\tau, y)$$

Similarly for a household choosing private school r:

$$\tau_r(y) = \arg\max_{\tau} V_r(\tau, y)$$

$$V_r^p(y) = \max_{\tau} V_r(\tau, y)$$

Lemma 3. *τ_u, $V_u^p(y)$, $\tau_r(y)$ and $V_r^p(y)$ are given by:*

$$\tau_u = \frac{\beta Y - \alpha \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}{Y(\alpha+\beta)}$$

$$V_u^p(y) = \frac{\alpha^\alpha \beta^\beta}{(\alpha+\beta)^{\alpha+\beta}} \left(\frac{y}{Y}\right)^\alpha \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\alpha+\beta}$$

$$\tau_r(y) = \frac{\beta Y y - (\beta Y \gamma_r + \alpha(y + Y\gamma_r)) \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}{Y(\alpha+\beta)(y+Y\gamma_r)}$$

$$V_r^p(y) = \frac{\alpha^\alpha \beta^\beta \gamma_r^\beta y^{\alpha+\beta} \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta} Y^\alpha (y+Y\gamma_r)^\beta}$$

Proof. First order condition in the definition of τ_u is:

$$\tau(\alpha+\beta)Yy = (\beta Y - \alpha \sum_{r=1}^{R} \frac{b_r}{\gamma_r})y$$

which implies the closed form expression for τ_u. Substituting τ_u into c_u and q_u in the Cobb-Douglas utility function gives the expression for $V_u^p(y)$.

Similarly first order condition in the definition of $\tau_r(y)$ is:

$$(\alpha(y+Y\gamma_r)Y + \beta Y y + \beta Y^2 \gamma_r)\tau_r(y) = \beta Y y - (\beta Y \gamma_r + \alpha(y+Y\gamma_r)) \sum_{r=1}^{R} \frac{b_r}{\gamma_r}$$

which implies the closed form expression for $\tau_r(y)$. Substituting $\tau_r(y)$ into c_r and q_r in the Cobb-Douglas utility function gives the expression for $V_r^p(y)$. □

Proposition 3. *The following holds:*

- *For households with income $y \leq \frac{Y\gamma_1}{\gamma_1 - 1}$, $V_u^p(y) \geq V_1^p(y)$.*
- *If $\gamma_r < \gamma_{r'}$, then $V_r^p(y) < V_{r'}^p(y)$.*

Proof. Therefore if $y \leq \frac{Y\gamma_1}{\gamma_1 - 1}$, then $y \leq \frac{Y\gamma_1}{\gamma_1 - 1}$ which implies:

$$y(\gamma_1 - 1) \leq Y\gamma_1$$

$$\Rightarrow \frac{\gamma_1 y}{y + Y\gamma_1} \leq 1$$

$$\Rightarrow \frac{\gamma_1^\beta y^\beta}{(y + Y\gamma_1)^\beta} \leq 1$$

$$\Rightarrow \frac{\beta^\beta \gamma_1^\beta y^\beta \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^\beta}{(\alpha+\beta)^\beta (y + Y\gamma_1)^\beta} \leq \frac{\beta^\beta \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^\beta}{(\alpha+\beta)^\beta}$$

$$\Rightarrow \frac{\alpha^\alpha \beta^\beta \gamma_1^\beta y^{\alpha+\beta} \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta} Y^\alpha (y + Y\gamma_1)^\beta} \leq \frac{\alpha^\alpha \beta^\beta}{(\alpha+\beta)^{\alpha+\beta}} \left(\frac{y}{Y}\right)^\alpha \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\alpha+\beta}$$

$$\Rightarrow V_1^p(y) \leq V_u^p(y)$$

Now let us prove the second claim.

$$\gamma_r y < \gamma_{r'} y$$

$$\Rightarrow \gamma_r y + Y\gamma_r\gamma_{r'} < \gamma_{r'} y + Y\gamma_r\gamma_{r'}$$

$$\Rightarrow \gamma_r (y + Y\gamma_{r'}) < \gamma_{r'} (y + Y\gamma_r)$$

$$\Rightarrow \left(\frac{\gamma_r}{y + Y\gamma_r}\right)^\beta < \left(\frac{\gamma_{r'}}{y + Y\gamma_{r'}}\right)^\beta$$

$$\Rightarrow \frac{\alpha^\alpha \beta^\beta \gamma_r^\beta y^{\alpha+\beta} \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta} Y^\alpha (y + Y\gamma_r)^\beta} < \frac{\alpha^\alpha \beta^\beta \gamma_{r'}^\beta y^{\alpha+\beta} \left(Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\alpha+\beta}}{(\alpha+\beta)^{\alpha+\beta} Y^\alpha (y + Y\gamma_{r'})^\beta}$$

$$\Rightarrow V_r^p(y) < V_{r'}^p(y)$$

$\square$

Proposition 4. *The following holds:*

- *If* $\gamma_r < \gamma_{r'}$, *then* $\tau_r(y) > \tau_{r'}(y)$
- $\tau_u > \tau_1$

Proof. Choose arbitrary private schools r and r' such that $\gamma_r < \gamma_{r'}$. Notice that:

$$\beta y Y \gamma_r < \beta y Y \gamma_{r'}$$

$$\Rightarrow ((\alpha+\beta)Y\gamma_{r'} + \beta y)Y\gamma_r < ((\alpha+\beta)Y\gamma_r + \beta y)Y\gamma_{r'}$$

$$\Rightarrow ((\alpha+\beta)(y+Y\gamma_{r'}) - \alpha y)Y\gamma_r < ((\alpha+\beta)(y+Y\gamma_r) - \alpha y)Y\gamma_{r'}$$

$$\Rightarrow \alpha y Y\gamma_{r'} + (\alpha+\beta)(y+Y\gamma_{r'})Y\gamma_r < \alpha y Y\gamma_r + (\alpha+\beta)(y+Y\gamma_r)Y\gamma_{r'}$$

$$\Rightarrow \alpha y(y+Y\gamma_{r'}) + (\alpha+\beta)(y+Y\gamma_{r'})Y\gamma_r < \alpha y(y+Y\gamma_r) + (\alpha+\beta)(y+Y\gamma_r)Y\gamma_{r'}$$

$$\Rightarrow (\beta Y\gamma_r + \alpha(y+Y\gamma_r))(y+Y\gamma_{r'}) < (\beta Y\gamma_{r'} + \alpha(y+Y\gamma_{r'}))(y+Y\gamma_r)$$

$$\Rightarrow -(\beta Y\gamma_r + \alpha(y+Y\gamma_r))(y+Y\gamma_{r'})\sum_{r=1}^{R}\frac{b_r}{\gamma_r} > -(\beta Y\gamma_{r'} + \alpha(y+Y\gamma_{r'}))(y+Y\gamma_r)\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$

$$\Rightarrow \beta Y y(y+Y\gamma_{r'}) - (\beta Y\gamma_r + \alpha(y+Y\gamma_r))(y+Y\gamma_{r'})\sum_{r=1}^{R}\frac{b_r}{\gamma_r} >$$
$$\beta Y y(y+Y\gamma_r) - (\beta Y\gamma_{r'} + \alpha(y+Y\gamma_{r'}))(y+Y\gamma_r)\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$

$$\Rightarrow \frac{\beta Y y - (\beta Y\gamma_r + \alpha(y+Y\gamma_r))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y+Y\gamma_r} > \frac{\beta Y y - (\beta Y\gamma_{r'} + \alpha(y+Y\gamma_{r'}))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y+Y\gamma_{r'}}$$

$$\Rightarrow \frac{\beta Y y - (\beta Y\gamma_r + \alpha(y+Y\gamma_r))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{Y(\alpha+\beta)(y+Y\gamma_r)} > \frac{\beta Y y - (\beta Y\gamma_{r'} + \alpha(y+Y\gamma_{r'}))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{Y(\alpha+\beta)(y+Y\gamma_{r'})}$$

$$\Rightarrow \tau_r(y) > \tau_{r'}(y)$$

which establishes the result since r and $r^{'}$ are arbitrary and $\gamma_1 < \gamma_2 < ... < \gamma_R$.

Now let us prove $\tau_u > \tau_1$. Notice that:

$$\beta Y^2\gamma_1 > -\beta Y\gamma_1\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$

$$\Rightarrow \beta Yy + \beta Y^2\gamma_1 - \alpha(y+Y\gamma_1)\sum_{r=1}^{R}\frac{b_r}{\gamma_r} > \beta Yy - \beta Y\gamma_1\sum_{r=1}^{R}\frac{b_r}{\gamma_r} - \alpha(y+Y\gamma_1)\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$

$$\beta Y(y+Y\gamma_1) - \alpha(y+Y\gamma_1)\sum_{r=1}^{R}\frac{b_r}{\gamma_r} > \beta Yy - (\beta Y\gamma_1 + \alpha(y+Y\gamma_1))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$

$$\beta Y - \alpha\sum_{r=1}^{R}\frac{b_r}{\gamma_r} > \frac{\beta Yy - (\beta Y\gamma_1 + \alpha(y+Y\gamma_1))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y+Y\gamma_1}$$

$$\Rightarrow \frac{\beta Y - \alpha\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{Y(\alpha+\beta)} > \frac{\beta Yy - (\beta Y\gamma_1 + \alpha(y+Y\gamma_1))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{Y(\alpha+\beta)(y+Y\gamma_1)}$$

$$\Rightarrow \tau_u > \tau_1$$

□

Definition 5. *$\widehat{\tau}_{rr'}(y)$ is defined as the tax rate at which household with income y is indifferent between private schools r and $r^{'}$ where $\gamma_r < \gamma_{r'}$, i.e:*

$$V_r(\widehat{\tau}_{rr'}(y), y) = V_{r'}(\widehat{\tau}_{rr'}(y), y)$$

Lemma 4. *For those households with income $y \geq \frac{(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}}}$ there exists a unique positive $\widehat{\tau}_{rr'}(y)$ which is given by:*

$$\widehat{\tau}_{rr'}(y) = \frac{y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}}) - (\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}}) + (\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})Y}$$

Proof.

$$
\begin{aligned}
&V_r(\widehat{\tau}_{rr'}(y),y) = V_{r'}(\widehat{\tau}_{rr'}(y),y)\\
\Rightarrow\ &((1-\widehat{\tau}_{rr'})y)^{\alpha}\left(\widehat{\tau}_{rr'}Y+\sum_{r=1}^{R}\frac{b_r}{\gamma_r}\right)^{\beta}\\
=\ &\left((1-\widehat{\tau}_{rr'})y-\widehat{\tau}_{rr'}Y\gamma_r-\gamma_r\sum_{r=1}^{R}\frac{b_r}{\gamma_r}\right)^{\alpha}\left(\gamma_r\widehat{\tau}_{rr'}Y+\gamma_r\sum_{r=1}^{R}\frac{b_r}{\gamma_r}\right)^{\beta}\\
\Rightarrow\ &[y-\gamma_r\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-\widehat{\tau}_{rr'}(y+Y\gamma_r)]\gamma_r^{\frac{\beta}{\alpha}} = [y-\gamma_{r'}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-\widehat{\tau}_{rr'}(y+Y\gamma_{r'})]\gamma_{r'}^{\frac{\beta}{\alpha}}\\
\Rightarrow\ &y(\gamma_r^{\frac{\beta}{\alpha}}-\gamma_{r'}^{\frac{\beta}{\alpha}})+(\gamma_r^{\frac{\alpha+\beta}{\alpha}}-\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r} = \widehat{\tau}_{rr'}[\gamma_r^{\frac{\beta}{\alpha}}(y+Y\gamma_r)-\gamma_{r'}^{\frac{\beta}{\alpha}}(y+Y\gamma_{r'})]\\
\Rightarrow\ &\widehat{\tau}_{rr'}(y) = \frac{y(\gamma_{r'}^{\frac{\beta}{\alpha}}-\gamma_r^{\frac{\beta}{\alpha}})-(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}}-\gamma_r^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y(\gamma_{r'}^{\frac{\beta}{\alpha}}-\gamma_r^{\frac{\beta}{\alpha}})+(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}}-\gamma_r^{\frac{\alpha+\beta}{\alpha}})Y}
\end{aligned}
$$

□

Assumption 7. *For two private schools r and r' with $\gamma_r < \gamma_{r'}$, the following holds:*

$$
\gamma_{r'}^{\frac{\beta}{\alpha}}(\beta Y\gamma_{r'}-(\alpha+\beta)\gamma_r\sum_{r=1}^{R}\frac{b_r}{\gamma_r})+(\alpha\gamma_r^{\frac{\alpha+\beta}{\alpha}}+\beta\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-\gamma_r Y[(\alpha+\beta)\gamma_{r'}^{\frac{\beta}{\alpha}}-\alpha\gamma_r^{\frac{\beta}{\alpha}}]\le 0
$$

Proposition 5. *If assumption 7 holds, then $\tau_r(y)\le\widehat{\tau}_{rr'}(y)$.*

Proof. Assumption 7 implies:

$$
Y\gamma_{r'}^{\frac{\beta}{\alpha}}(\beta Y\gamma_{r'}-(\alpha+\beta)\gamma_r\sum_{r=1}^{R}\frac{b_r}{\gamma_r})\le -Y(\alpha\gamma_r^{\frac{\alpha+\beta}{\alpha}}+\beta\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}+Y^2\gamma_r[(\alpha+\beta)\gamma_{r'}^{\frac{\beta}{\alpha}}-\alpha\gamma_r^{\frac{\beta}{\alpha}}]
$$

This implies:

$$
\begin{aligned}
&Y\gamma_{r'}^{\frac{\beta}{\alpha}}(\beta Y\gamma_{r'}-(\alpha+\beta)\gamma_r\sum_{r=1}^{R}\frac{b_r}{\gamma_r})+\alpha Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}+\beta Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-\beta Y^2\gamma_r^{\frac{\alpha+\beta}{\alpha}}\le\\
&-Y(\alpha\gamma_r^{\frac{\alpha+\beta}{\alpha}}+\beta\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}+Y^2\gamma_r[(\alpha+\beta)\gamma_{r'}^{\frac{\beta}{\alpha}}-\alpha\gamma_r^{\frac{\beta}{\alpha}}]+\alpha Y\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-\\
&\alpha Y\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}+\alpha Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}+\beta Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-\beta Y^2\gamma_r^{\frac{\alpha+\beta}{\alpha}}
\end{aligned}
$$

Remember that $\widehat{\tau}_{rr'}(y)$ is defined for those incomes that satisfy $y \geq \frac{(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}}}$.

Therefore the last inequality can also be written as:

$$\beta Y^2(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}}) - (\alpha+\beta)\gamma_r Y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$
$$\leq \alpha Y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}})y + (\alpha+\beta)\gamma_r Y^2(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}}) - (\alpha+\beta)\gamma_r Y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$

This implies:

$$\beta Y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}})y^2 + \beta Y^2(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})y - (\alpha+\beta)\gamma_r Y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}})y\sum_{r=1}^{R}\frac{b_r}{\gamma_r} -$$
$$(\alpha+\beta)\gamma_r Y^2(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})y\sum_{r=1}^{R}\frac{b_r}{\gamma_r} \leq Y(\alpha+\beta)(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}})y^2 + (\alpha+\beta)\gamma_r Y^2(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}})y$$
$$(\alpha+\beta)Y(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})y\sum_{r=1}^{R}\frac{b_r}{\gamma_r} - (\alpha+\beta)Y^2\gamma_r(\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}$$

This inequality is equivalent to:

$$\frac{\beta Yy - (\beta Y\gamma_r + \alpha(y + Y\gamma_r))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{Y(\alpha+\beta)(y+Y\gamma_r)} \leq \frac{y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}}) - (\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y(\gamma_{r'}^{\frac{\beta}{\alpha}} - \gamma_r^{\frac{\beta}{\alpha}}) + (\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}} - \gamma_r^{\frac{\alpha+\beta}{\alpha}})Y}$$

Therefore;

$$\tau_r(y) \leq \widehat{\tau}_{rr'}(y)$$

$\square$

Proposition 5 is illustrated in Figure 2.3 for the case of two private schools.

Definition 6. *$\widehat{\tau}_{ur}(y)$ is defined as the tax rate at which household with income y is indifferent between public school and private school r, i.e:*

$$V_u(\widehat{\tau}_{ur}(y), y) = V_r(\widehat{\tau}_{ur}(y), y)$$

Lemma 5. *For those households with income* $y \geq \frac{\gamma_r^{\frac{\alpha+\beta}{\alpha}} \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}{\gamma_r^{\frac{\beta}{\alpha}} - 1}$ *there exists a unique positive* $\widehat{\tau}_{ur}(y)$ *which is given by:*

$$\widehat{\tau}_{ur}(y) = \frac{y(\gamma_r^{\frac{\beta}{\alpha}} - 1) - \gamma_r^{\frac{\alpha+\beta}{\alpha}} \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}{y(\gamma_r^{\frac{\beta}{\alpha}} - 1) + Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}}$$

Proof.

$$\begin{aligned}
V_u(\widehat{\tau}_{ur}(y), y) &= V_r(\widehat{\tau}_{ur}(y), y) \\
(1 - \widehat{\tau}_{ur}(y))^{\alpha} y^{\alpha} \left(\widehat{\tau}_{ur}(y)Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\beta} &= \\
\left[(1 - \widehat{\tau}_{ur}(y))y - \gamma_r \left(\widehat{\tau}_{ur}(y)Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)\right]^{\alpha} \gamma_r^{\beta} \left(\widehat{\tau}_{ur}(y)Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)^{\beta} \\
\Rightarrow (1 - \widehat{\tau}_{ur}(y))y &= \left[(1 - \widehat{\tau}_{ur}(y))y - \gamma_r \left(\widehat{\tau}_{ur}(y)Y + \sum_{r=1}^{R} \frac{b_r}{\gamma_r}\right)\right] \gamma_r^{\frac{\beta}{\alpha}} \\
\Rightarrow \widehat{\tau}_{ur}(y) &= \frac{y(\gamma_r^{\frac{\beta}{\alpha}} - 1) - \gamma_r^{\frac{\alpha+\beta}{\alpha}} \sum_{r=1}^{R} \frac{b_r}{\gamma_r}}{y(\gamma_r^{\frac{\beta}{\alpha}} - 1) + Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}}
\end{aligned}$$

□

Assumption 8. *The following holds for any* r*:*

$$\alpha(Y+1)(\gamma_r^{\frac{\beta}{\alpha}} - 1)\frac{\gamma_1^2}{(\gamma_1 - 1)^2} + (\gamma_r^{\frac{\beta}{\alpha}} - 1)((\alpha+\beta)Y + \alpha + \beta)\frac{\gamma_r\gamma_1}{\gamma_1 - 1}$$
$$- \gamma_r^{\frac{\beta}{\alpha}} \left((\alpha+\beta)\sum_{r=1}^{R} \frac{b_r}{\gamma_r} + \beta Y - \alpha\right)\frac{\gamma_r\gamma_1}{\gamma_1 - 1} \leq (\alpha+\beta)\gamma_r^{\frac{2\alpha+\beta}{\alpha}} \left[\sum_{r=1}^{R} \frac{b_r}{\gamma_r} - 1\right]$$

Proposition 6. *If assumption 8 holds, then* $\widehat{\tau}_{ur}(y) \leq \tau_r(y)$ *for households with* $y \leq \frac{Y\gamma_1}{\gamma_1 - 1}$.

Proof. Assumption 8 together with $y \leq \frac{Y\gamma_1}{\gamma_1 - 1}$ implies:

$$\alpha(Y+1)(\gamma_r^{\frac{\beta}{\alpha}} - 1)y^2 + \Bigg[(\gamma_r^{\frac{\beta}{\alpha}} - 1)Y^2(\alpha+\beta)\gamma_r - Y(\alpha+\beta)\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r} - \beta Y^2\gamma_r^{\frac{\alpha+\beta}{\alpha}} + \alpha Y\gamma_r^{\frac{\alpha+\beta}{\alpha}} + \beta Y\gamma_r(\gamma_r^{\frac{\beta}{\alpha}} - 1) + \alpha(\gamma_r^{\frac{\beta}{\alpha}} - 1)Y\gamma_r\Bigg]y \leq Y^2(\alpha+\beta)\gamma_r^{\frac{2\alpha+\beta}{\alpha}}\left[\sum_{r=1}^{R}\frac{b_r}{\gamma_r} - 1\right]$$

This implies:

$$yY(\alpha+\beta)(y+Y\gamma_r)(\gamma_r^{\frac{\beta}{\alpha}} - 1) - Y(\alpha+\beta)(y+Y\gamma_r)\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r} \leq \beta Y(\gamma_r^{\frac{\beta}{\alpha}} - 1)y^2 + \beta Y^2\gamma_r^{\frac{\alpha+\beta}{\alpha}}y - (\beta Y\gamma_r + \alpha(y+Y\gamma_r))(\gamma_r^{\frac{\beta}{\alpha}} - 1)y - (\beta Y\gamma_r + \alpha(y+Y\gamma_r))Y\gamma_{r'}^{\frac{\alpha+\beta}{\alpha}}$$

This implies:

$$\frac{y(\gamma_r^{\frac{\beta}{\alpha}} - 1) - \gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y(\gamma_r^{\frac{\beta}{\alpha}} - 1) + Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}} \leq \frac{\beta Yy - (\beta Y\gamma_r + \alpha(y+Y\gamma_r))\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{Y(\alpha+\beta)(y+Y\gamma_r)}$$

Therefore:

$$\widehat{\tau}_{ur}(y) \leq \tau_r(y)$$

□

Assumption 9. *The following holds for any r:*

$$\gamma_r^{\frac{\alpha+\beta}{\alpha}}\beta Y \geq \alpha(\gamma_r^{\frac{\beta}{\alpha}} - 1)max\left\{\frac{Y\gamma_1}{\gamma_1 - 1}, \frac{\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{\gamma_r^{\frac{\beta}{\alpha}} - 1}\right\}$$

Proposition 7. *If assumption 9 holds, then $\widehat{\tau}_{ur}(y) \geq \tau_u$ for households with $y > \frac{Y\gamma_1}{\gamma_1 - 1}$*

Proof. For $\widehat{\tau}_{ur}(y)$ to be well defined, it is required that $y \geq \frac{\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{\gamma_r^{\frac{\beta}{\alpha}} - 1}$. For those households such that $y > max\left\{\frac{Y\gamma_1}{\gamma_1 - 1}, \frac{\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{\gamma_r^{\frac{\beta}{\alpha}} - 1}\right\}$, assumption 9 implies:

$$\gamma_r^{\frac{\alpha+\beta}{\alpha}}\beta Y \geq \alpha(\gamma_r^{\frac{\beta}{\alpha}} - 1)y$$

$$\gamma_r^{\frac{\alpha+\beta}{\alpha}}\beta Y\left[-\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-Y\right]\geq\alpha(\gamma_r^{\frac{\beta}{\alpha}}-1)y\left[-\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-Y\right]$$

This implies:

$$\alpha Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-Y(\alpha+\beta)\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-\beta Y^2\gamma_r^{\frac{\alpha+\beta}{\alpha}}\geq$$

$$\left[\beta Y(\gamma_r^{\frac{\beta}{\alpha}}-1)-\alpha(\gamma_r^{\frac{\beta}{\alpha}}-1)\sum_{r=1}^{R}\frac{b_r}{\gamma_r}-Y(\alpha+\beta)(\gamma_r^{\frac{\beta}{\alpha}}-1)\right]y$$

$$\Rightarrow\frac{y(\gamma_r^{\frac{\beta}{\alpha}}-1)-\gamma_r^{\frac{\alpha+\beta}{\alpha}}\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{y(\gamma_r^{\frac{\beta}{\alpha}}-1)+Y\gamma_r^{\frac{\alpha+\beta}{\alpha}}}\geq\frac{\beta Y-\alpha\sum_{r=1}^{R}\frac{b_r}{\gamma_r}}{Y(\alpha+\beta)}$$

$$\Rightarrow\widehat{\tau}_{ur}(y)\geq\tau_u$$

□

Theorem 2. *There exists a majority voting equilibrium and a decisive voter.*

Proof. Propositions 2- 7 imply preferences are single peaked over income tax rates. Rest follows from the theorem of Black (1948). □

Theorem 3. *If $\frac{\gamma_1}{\gamma_1-1}=\frac{Y+\sigma}{Y}$, then median income household is decisive voter.*

Proof. Household with income $Y\frac{\gamma_1}{\gamma_1-1}$ is indifferent between public school and private school type 1. Households with income below this cutoff chooses public school and votes for τ_u. I will prove that median income household's income is smaller than this cutoff. Let's denote median income with y_m. By O'Cinneide (1990),

$$|y_m-Y|\leq\sigma$$

$$\Rightarrow y_m\leq Y\frac{Y+\sigma}{Y}$$

$$\Rightarrow y_m \leq Y \frac{\gamma_1}{\gamma_1 - 1}$$

Therefore what is chosen by median income household always receives at least half of the votes. □

2.5 Conclusion

This paper provided a median voter theorem for an economy where heterogeneous income households can opt out of public education. Differently from Stiglitz (1974) and consistently with empirical evidence, I model private schools as monopolistically competitive firms with decreasing average costs over enrollment. In the model, there are a finite number of different quality private schools each having a different tuition. Public school spending is financed by income tax revenue collected from all households. The tax rate is determined by majority voting. In my model when income tax rate increases, enrollment increases in public schools and decreases in private schools. This increases tuition and per pupil spending in private schools. With Cobb-Douglas utility, income effect is dominated by substitution effect for households choosing private schools and they favor increases in tax rate up to some cutoff. Preferences over tax rates turn out to be single peaked and therefore a majority voting equilibrium exists. Moreover median income household is the decisive voter. These results hold for any income distribution function and any finite number of private schools.

In the model I concentrated on a single school district and a particular utility function. Extension of the results to an environment where utility function is arbitrary and households also choose among school districts would be interesting. Also, I have not estimated the theoretical model. The fit of the model to data is a relevant topic for future research too. Moreover, with the estimated model several private school voucher policies can be analyzed.

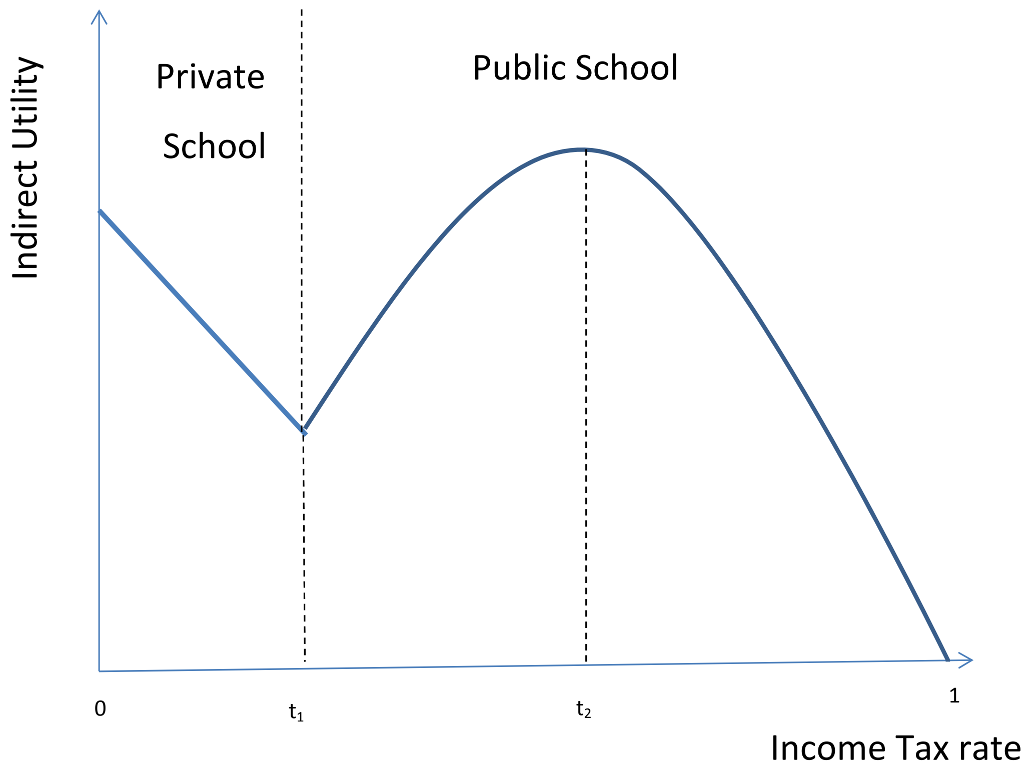

Figure 2.1: Double peaked preferences

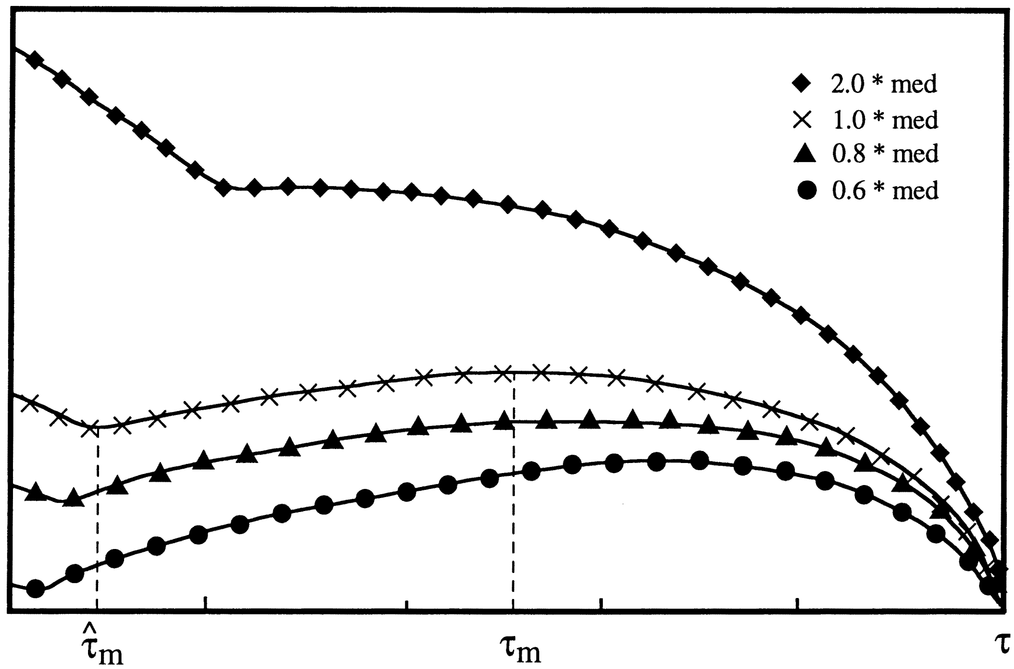

Figure 2.2: Preferences over tax rates in Glomm and Ravikumar (1998)

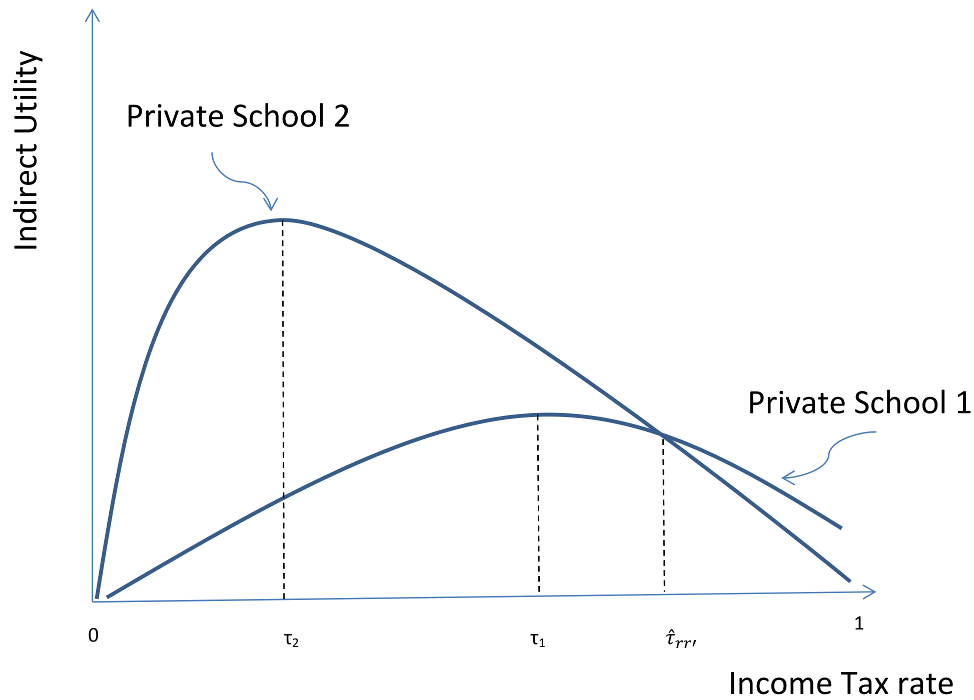

Figure 2.3: Illustration of proposition 5

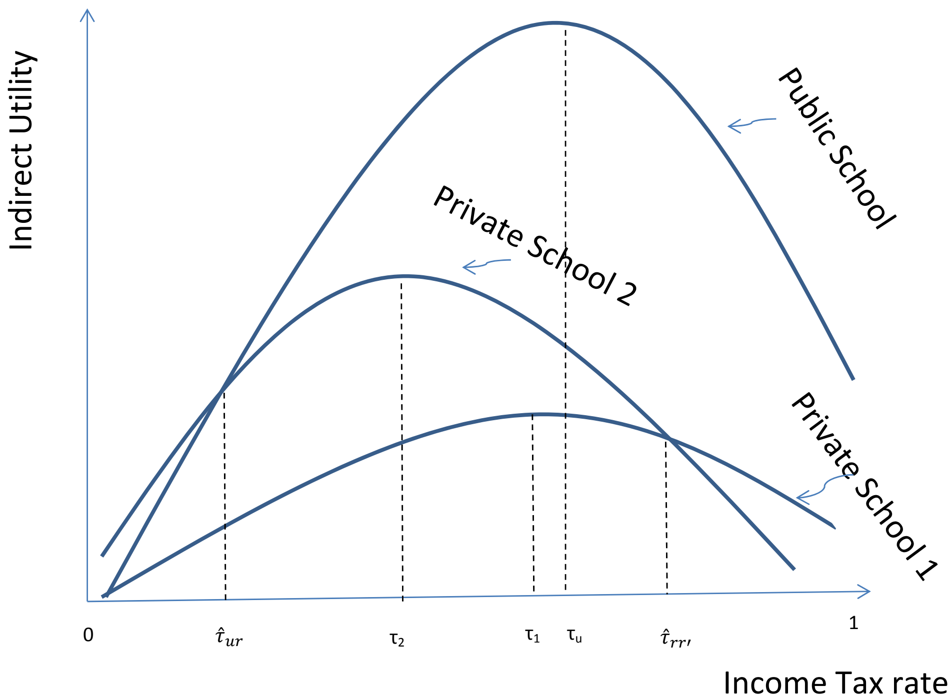

Figure 2.4: Illustration of proposition 6

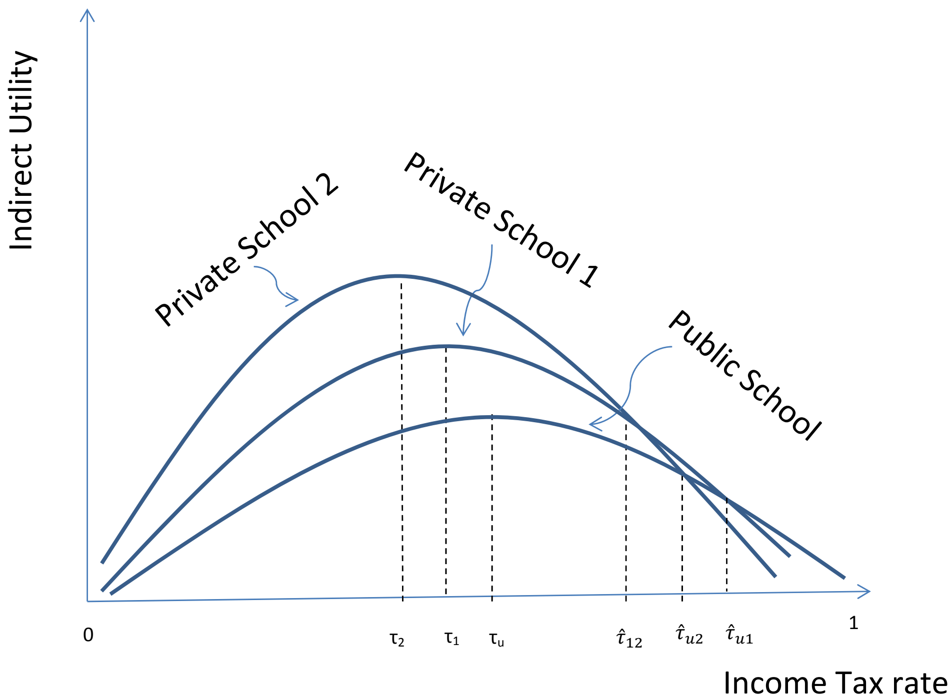

Figure 2.5: Illustration of proposition 7

APPENDIX
MATERIAL FOR CHAPTER 1

A.1 Data

Given the assumptions of the model, I choose to work with Rhode Island because:

- In the model, local property tax revenue is the most important component of local municipal revenues. In Rhode Island, local property tax revenue are on average $60\% - 70\%$ of the total municipal revenue.
- School choice is constrained by the boundary of municipality of residence in the model. In Rhode Island, inter-jurisdictional school choice programs are not allowed.
- In the model, households are not given the opportunity to choose a municipality outside the state. Net migration to Rhode Island between 2000-2004 Island is a small fraction, around 1.3%, of total population which demonstrates that households living in Rhode Island are naturally staying there. Rhode Island seems to be in a stationary equilibrium in this respect.
- Rhode Island is a small piece of land compared to other states which is important since I am not modeling tradeoffs related to transportation over space.

There are 14 municipalities belonging to Rhode Island in the original dataset for the year 2000.

Income distribution for municipalities and census tracts Income distribution data is from Census 2000 Table P52 for both municipalities and census tracts. What I call as municipality is called as Census Designated Place (CDP). It basically corresponds to towns and cities. I prefer to work with CDP's since each CDP has its own local government so has its own tax rate. Income distribution data shows the number of households that belong to a particular income bin for each CDP and each census tract. There are 16 income bins. I compute Gini index (G) of income for each CDP as follows:

$$G = 1 - \frac{\sum_{i=1}^{n} f(y_i)(R_{i-1} + R_i)}{R_n}$$

where $f(y)$ is the discrete probability distribution of income with n observations indexed by y_i and $R_i = \sum_{j=1}^{i} f(y_j)y_j$ with $R_0 = 0$.

S in Table 1.1 is computed as follows. First let's define within municipality variance of income (WG) and between municipality variance of income (BG):

$$BG = \sum_{m=1}^{M} \lambda_m \ln \frac{\overline{y}_m}{\overline{y}}$$

$$WG = \Sigma_{m=1}^{M} \lambda_m \int_y N_m \frac{y}{\overline{y}_m} \ln \frac{y}{\overline{y}_m} f_m(y) dy$$

where:

- λ_m is the income share of municipality m
- $\overline{y}_m$ is the mean income in municipality m
- $\overline{y}$ is the mean income in the society

- N_m is the measure of households living in municipality m

S is computed as:

$$S = \frac{WG}{WG + BG}$$

Median income for CDP's is from Census 2000 Table P53.

House value distribution The data of house value distribution for each CDP is the sum of owner occupied housing value distribution (Census 2000 Table H84) and renter occupied housing contract rent distribution (Census 2000 Table H54). The data comes in terms of value or contract rent intervals. I use equation (1.17) to convert contract rent into value and then I merge the two datasets to come up with the eventual house value distribution used in the computation which includes both owner and renter occupied housing units. In the dataset, there are 24 value intervals for each municipality. I decrease this to 6 by combining 4 consecutive intervals into one.

Joint distribution of household income and contract rent as a percentage of household income Census 2000 Table H73 gives joint distribution of household income and gross rent as a percentage of household income for renter occupied housing. Since gross rent includes also utilities, I subtract 6% gross rent percentage to obtain contract rent as a percentage of household income following Davis and Ortalo-Magne (2011) who founds that utilities account for 6% of household income. Since Table H73 is for renter occupied housing, I also consider Census

2000 Table HCT17 which provides joint distribution of household income and value of house occupied for owners. Using (1.17), I convert house values in Table HCT17 into contract rent and then merge this with Table H73 to obtain the data used in Figures 1.14 through 1.18.

Wealth distribution across municipalities and census tracts Census 2000 Table HCT17 provides joint distribution of household income and value of housing owned for both CDP's and census tracts. As mentioned in the introduction, I define wealth as the sum of annual income and value of housing owned.

Public spending per household, residential property tax rate and state aid This data comes from Rhode Island Department of Revenue for each municipality. In order to find per household numbers for each municipality, I divide by the total number of households in each municipality.

A.2 Grouping municipalities via hierarchical clustering

In the original dataset there are 14 municipalities in Rhode Island which are Barrington, Bristol, Central Falls, Cranston, East Providence, Narragansett, Newport, North Providence, Pawtucket, Providence, Tiverton, Warwick, West Warwick and Woonsocket. Because of computational difficulties, I am grouping these 14 municipalities into 5 using Hierarchical Clustering Method as explained in Kaufman and Rousseeuw (1990). It should be noted that Hierarchical Clustering is a path independent method compared to other data mining techniques such as K-Means

Clustering. Therefore for hierarchical clustering one does not have to make an initial guess. I cluster these municipalities so as to maximize the similarity within a group with respect to residential property tax rate, net state aid per household and house supply. Note that these variables are the only exogenous variables regarding municipalities.

How should the number of clusters be determined? I determine the optimal number of clusters by analyzing the change in SSE (sum of squared error with respect to ward metric) that results from adding a municipality to a group at each level of hierarchy. I plot number of clusters against the corresponding SSE in Figure A.1. As the figure suggests, there is a jump in SSE when number of clusters decreases from 4 to 3. Therefore 4 clusters seems like a natural choice. But just looking SSE's may be misleading. Another commonly used metric is to compare Silhouette coefficients for different number of clusters. I also compute Silhouette coefficients for each number of clusters. Higher values of Silhouette coefficient means better approximation. When there are 4 and 5 clusters, Silhouette coefficient is 0.7 and 0.72 respectively. Given that SSE is slightly smaller under 5 clusters compared to 4, I simply set the number of clusters to 5.

One natural question is which municipality is in which cluster? I provide the answer in Table A.1. It should ne noted that the biggest municipality in the dataset (Providence) is itself a cluster.

Figures A.2 through A.9 plot Facts 1 to 3 using original data for 14 munici-

Table A.1: Hierarchical clustering result

Municipality	Cluster
Barrington	III
Bristol	III
Central Falls	III
Cranston	V
East Providence	II
Narragansett	III
New Port	III
North Providence	I
Pawtucket	V
Providence	IV
Tiverton	III
Warwick	V
West Warwick	I
Woonsocket	II

palities before clustering. Facts 4 to 6 are given in Table A.2.

Table A.2: Clustered data vs. original data w.r.t. other facts

Correlations	Clustered Data	Original Data
Med. Income & Median House Value	0.72	0.66
Med. House Value & Per HH Public Spend.	0.55	0.65
Med. Income & Gini Index of Income	-0.59	-0.55

A.3 Computational algorithm

The computational procedure used to solve the model is a multivariate bisection method inserted in a nested fixed algorithm following closely Nechyba (1999) and it consists of one inner loop for finding house values p_{mh} and one outer loop for finding per pupil public education spending E_m as described below:

1. Create a grid of probabilities with 100 elements.

2. Guess an initial E_m^0 for each municipality.

   - Guess p_{mh}^0.

   - In the first iteration set $ED_{mh}^{-1} = 0$ and $p_{mh}^{-1} = p_{mh}^{-2} = p_{mh}^0$ where ED_{mh}^{-1}, p_{mh}^{-1}, p_{mh}^{-2} stand for excess demand in the previous iteration, house value in the previous iteration and two iterations prior to the current iteration respectively.

   - Solve household's first stage optimization problem (1.4) at each possible pair of house type alternatives and at each point in the grid of lotteries.[1] Since there are 30 house types that makes 435 possible combinations with two house types in each combination. Since there 100 points in the probability grid, that requires solving the problem for an individual at $435 * 100 = 43500$ different points. Consider an household choosing

[1] As noted in the model section, an arbitrary individual will at most randomize between two pairs of alternatives regardless of the number of alternatives.

among alternatives mh and $m'h'$. Then the solution to (1.4) under the utility specification given in the calibration section implies:

$$c_{mh} = \frac{(y - (\frac{1}{21} + \tau_{m'})p_{m'h'})\pi_{m'h'} + (y - (\frac{1}{21} + \tau_m)p_{mh})\pi_{mh}}{\pi_{mh} + (\frac{q_{mh}}{q_{m'h'}})^{\frac{1}{\alpha-1}}(\frac{\ln E^0_{mh}}{\ln E^0_{m'h'}})^{\frac{\gamma}{\alpha-1}}\pi_{m'h'}}$$

$$c_{m'h'} = (\frac{q_{mh}}{q_{m'h'}})^{\frac{1}{\alpha-1}}(\frac{\ln E^0_{mh}}{\ln E^0_{m'h'}})^{\frac{\gamma}{\alpha-1}}c_{mh}$$

- Solve second stage optimization problem (1.5) and find the optimal probabilities for the household at each alternative pair $\{mh, m'h'\}$ and choose that probability and alternative pair that maximizes expected utility.

- Find excess demand ED^0_{mh} by using housing market clearing condition and update house values as follows:

  – If $ED^0_{mh} > 0$ and $ED^{-1}_{mh} > 0$, then set $p^0_{mh} = p^{-1}_{mh} + c * norm(ED^0)$.

  – If $ED^0_{mh} < 0$ and $ED^{-1}_{mh} < 0$, then set $p^0_{mh} = p^{-1}_{mh} - c * norm(ED^0)$.

  – If $ED^0_{mh} > 0$ and $ED^{-1}_{mh} \leq 0$, then set $p^0_{mh} = (p^{-1}_{mh} + p^{-2}_{mh})/2$.

  – If $ED^0_{mh} < 0$ and $ED^{-1}_{mh} \geq 0$, then set $p^0_{mh} = (p^{-1}_{mh} + p^{-2}_{mh})/2$.

  where $c = 100$ initially and it is multiplied by 0.999 whenever $\|ED^0\| > \|ED^{-1}\|$.

In words,

  – Increase prices when excess demand is positive

  – Decrease prices when excess demand is negative

  - Average prices of the last two iterations when excess demand changes sign.
  - Update c if prices are increased or decreased too much.
- Iterate in the inner loop until $\left|\frac{ED_{mh}}{\mu_{mh}}\right|$ is close to zero for each house type mh.

3. Then compute E_m for each municipality using local government budget (1.6).

4. Iterate until E_m converges for each municipality. The computational algorithm is summarized in Figure A.10.

On a computer with Intel Core i7 3.06 GHz processor and 9 GB Ram, this program takes between 120-130 hours to run completely for given parameters. I am using GPU programming in Matlab to compute the model. For GPU programming see Aldrich, Fernandez-Villaverde, Gallant, and Rubio-Ramirez (2010).

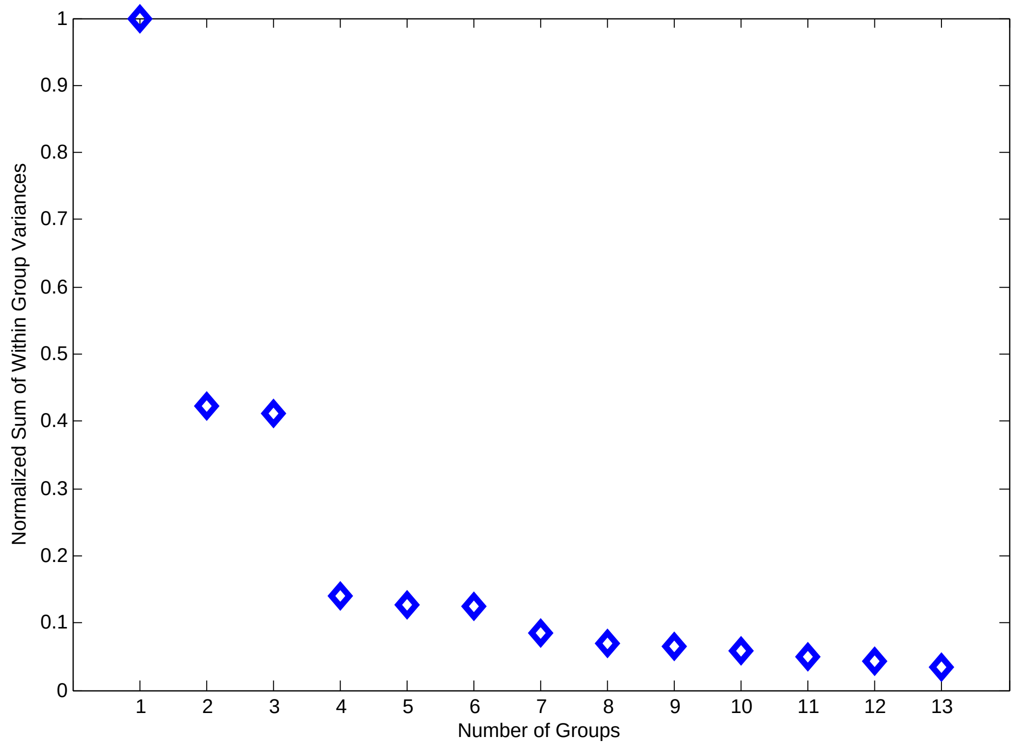

Figure A.1: Determining the optimal number of clusters

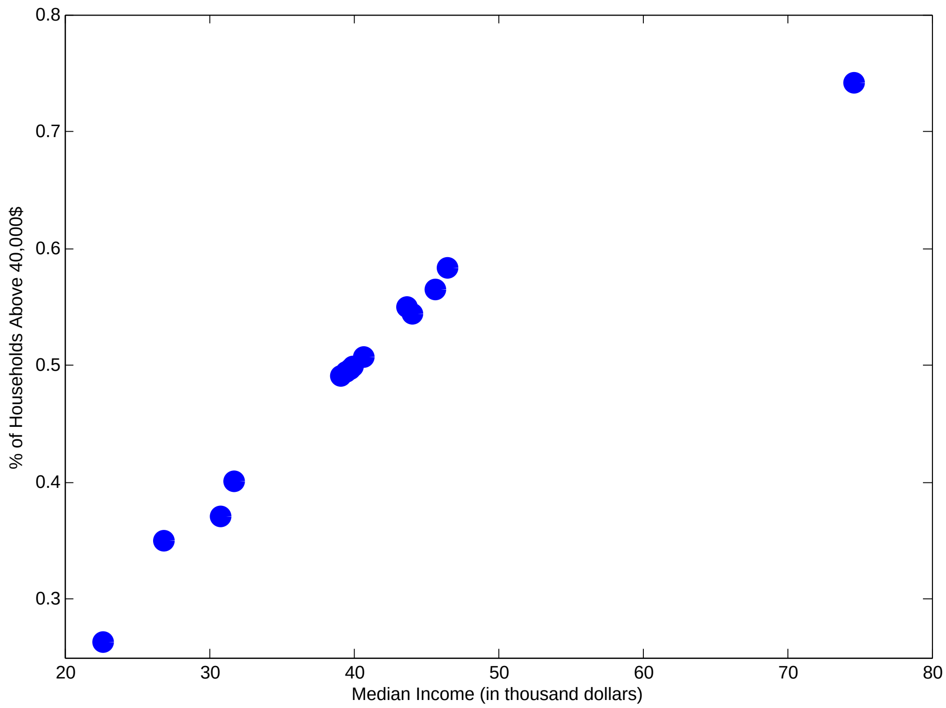

Figure A.2: Imperfect income sorting across Rhode Island's municipalities: Original data

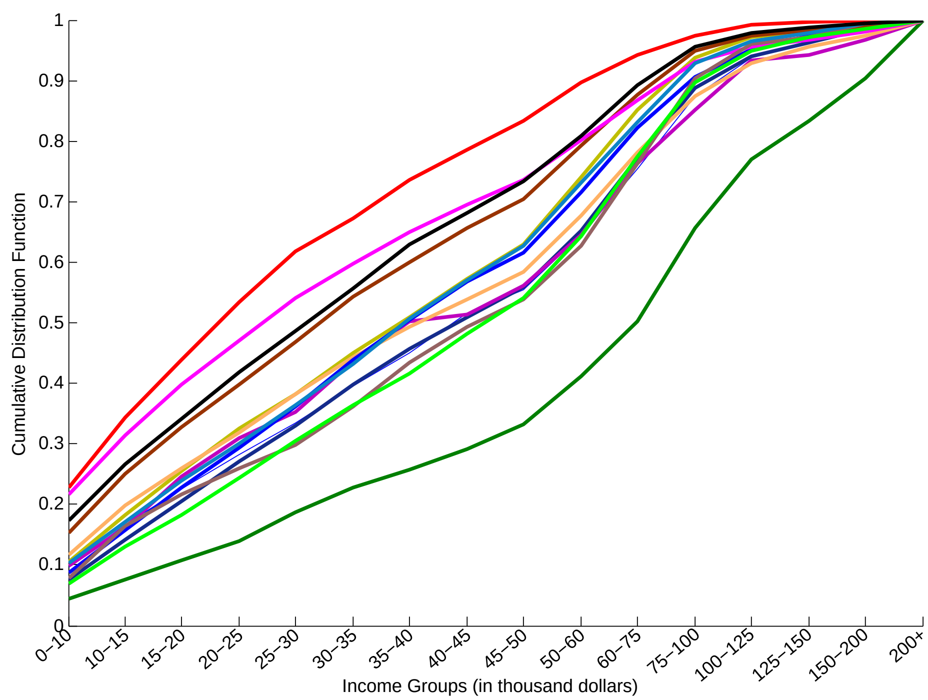

Figure A.3: Income distribution across Rhode Island's municipalities: Original data

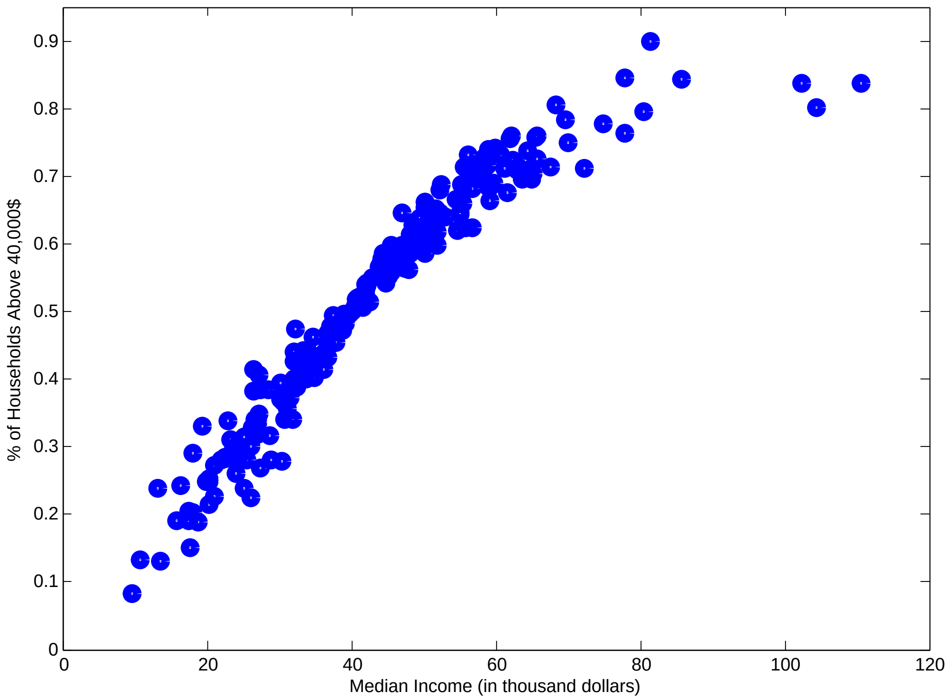

Figure A.4: Imperfect income sorting across Rhode Island's census tracts: Original data

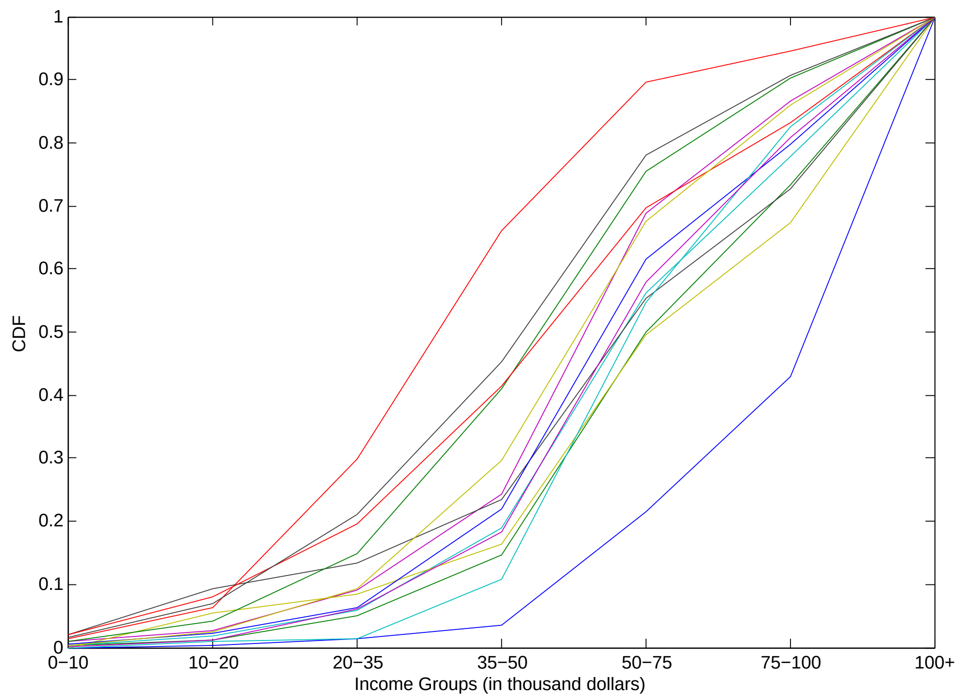

Figure A.5: Imperfect income sorting conditional on rent share in income less than 14%: Original data

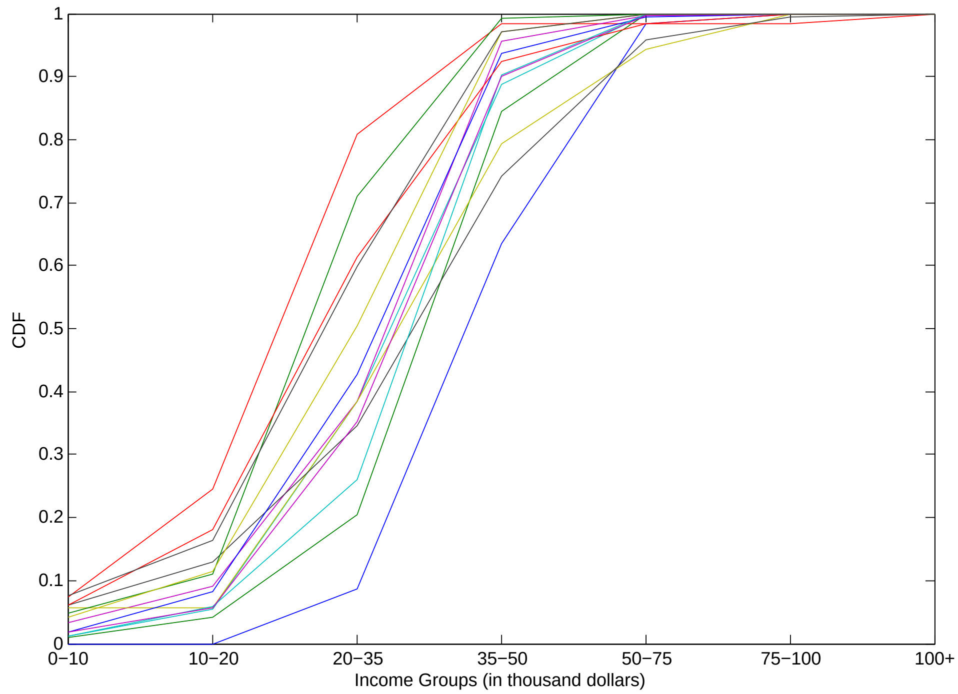

Figure A.6: Imperfect income sorting conditional on rent share in income between 14% and 19%: Original data

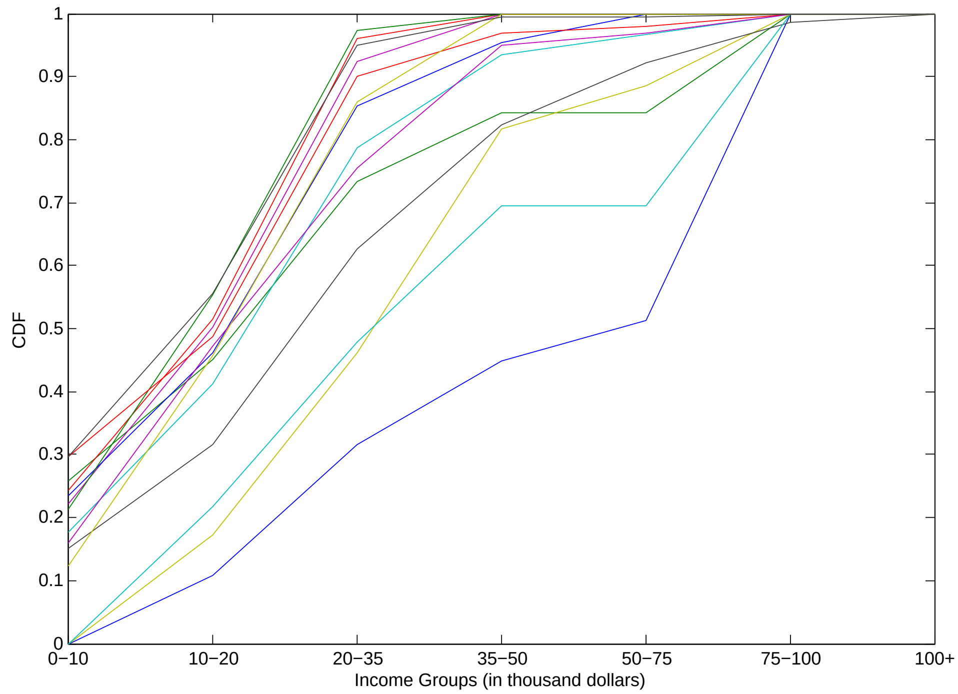

Figure A.7: Imperfect income sorting conditional on rent share in income between 19% and 24%: Original data

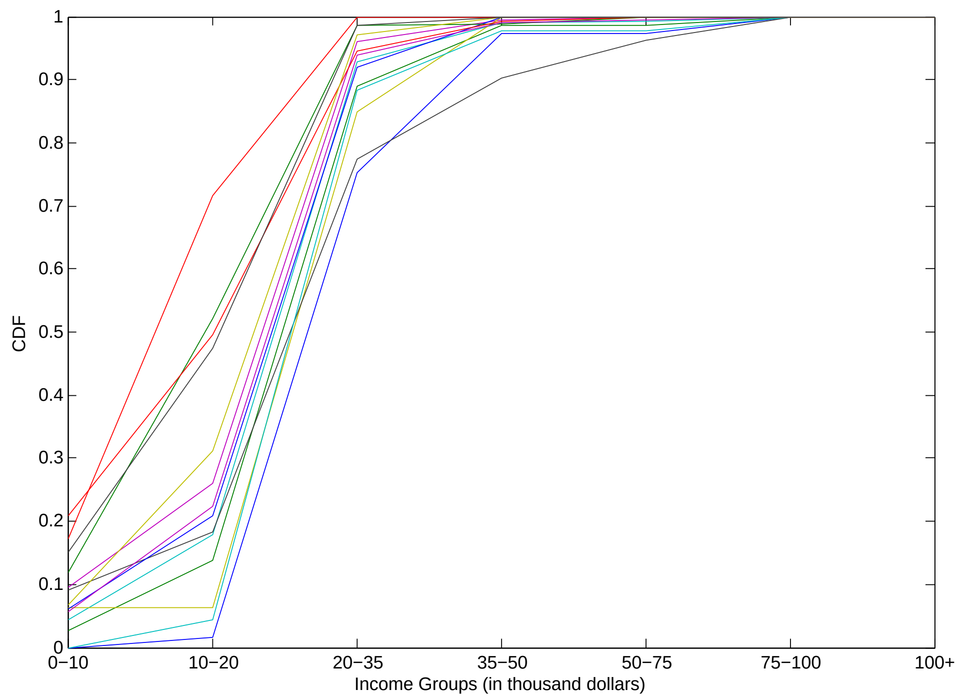

Figure A.8: Imperfect income sorting conditional on rent share in income between 24% and 29%: Original data

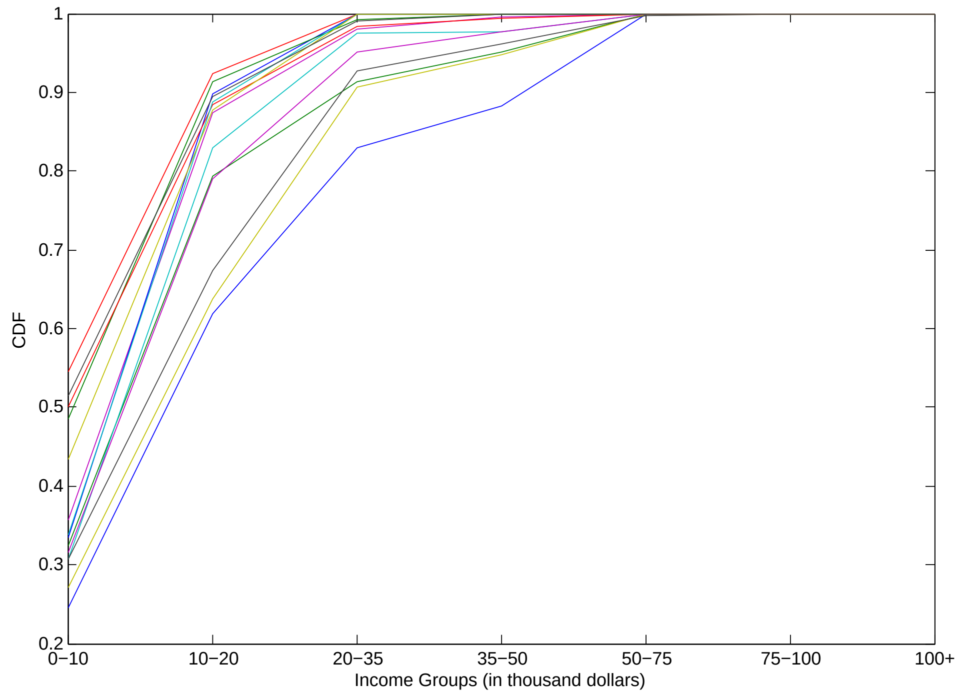

Figure A.9: Imperfect income sorting conditional on rent share in income more than 29%: Original data

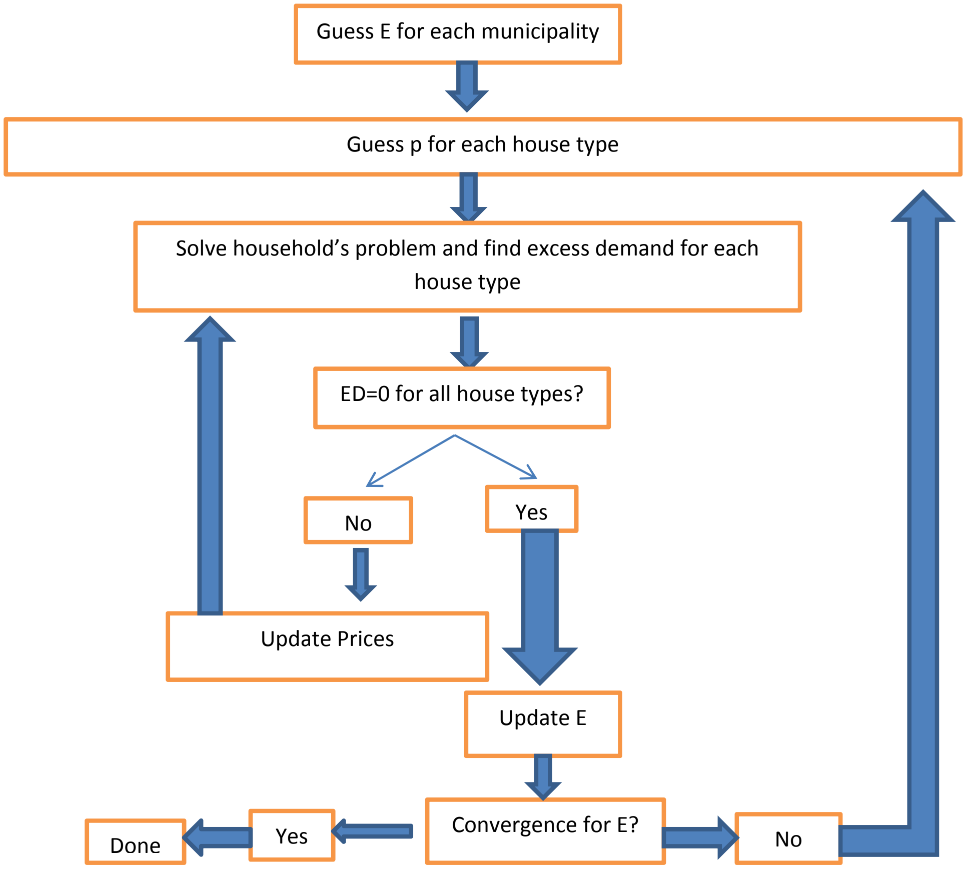

Figure A.10: Computational algorithm

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